Carnegie Mellon University

A Novel Field Method for the Flux Divergence

NORTHWEST GLACIOLOGISTS
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Background

- Mountain glaciers are changing rapidly
- We can measure surface elevation change for all glaciers on Earth (e.g., Hugonnet et al. 2021, Jakob & Gourmelen 2023)
- ***We need to translate distributed elevation change observations into climatic mass balance observations that can constrain global glacier models***

The *flux divergence* is a critical component needed to obtain the climatic mass balance

Climatic Mass Balance

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

Flux Divergence

Total Mass Balance

The *flux divergence* is a critical component needed to obtain the climatic mass balance

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

Climatic Mass Balance	Total Mass Balance	Flux Divergence
"Glacier mass change due to the climate" "Surface (+ internal) mass balance"	"Glacier surface elevation change" "Change in glacier thickness"	"Dynamic contribution to glacier surface change" "Elevation change from a difference in mass flux"

NOTE: the climatic mass balance and the total mass balance are equal glacier-wide, but vary spatially across the glacier

Climatic mass balance *requires* flux divergence without in-situ observations available

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

Climatic Mass Balance	Total Mass Balance	Flux Divergence
• Ablation stakes • Snow pits ***(in-situ)***	• DEM differencing ***(remote sensing)***	• Derived from ice thickness & velocity ***(remote sensing)*** • Field methods? ***(in-situ)***

We have no scaled constraints on modeled climatic mass balance which is crucial for process-based understanding of present and future glacier changes

Climatic mass balance *requires* flux divergence without in-situ observations available

This is what we want, globally…

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

…so we need this…

Climatic Mass Balance

- Ablation stakes
- Snow pits
  (in-situ)

Total Mass Balance

- DEM differencing
  (remote sensing)

Flux Divergence

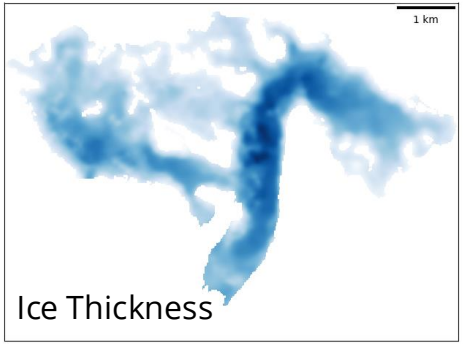
- Derived from ice thickness & velocity
  (remote sensing)
- Field methods?
  (in-situ)

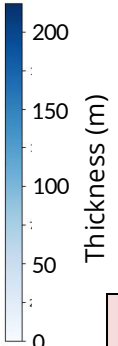
…for which ground-truth data is essential

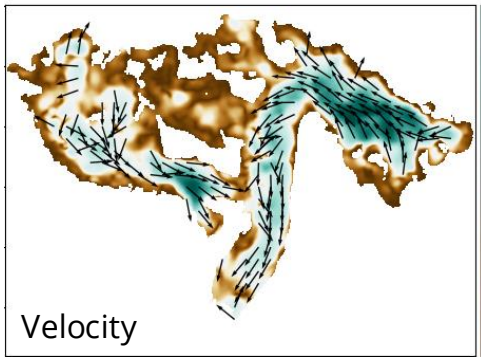
We have no scaled constraints on modeled climatic mass balance
based understanding of present and future glac

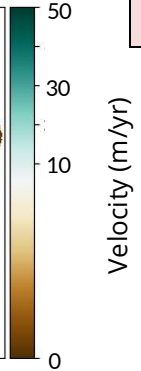
How do conventional remote-sensing approaches perform?

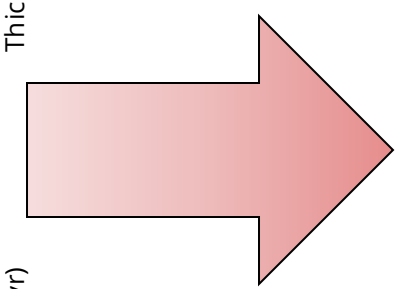
Remote sensing flux divergence is unreliable for point estimates



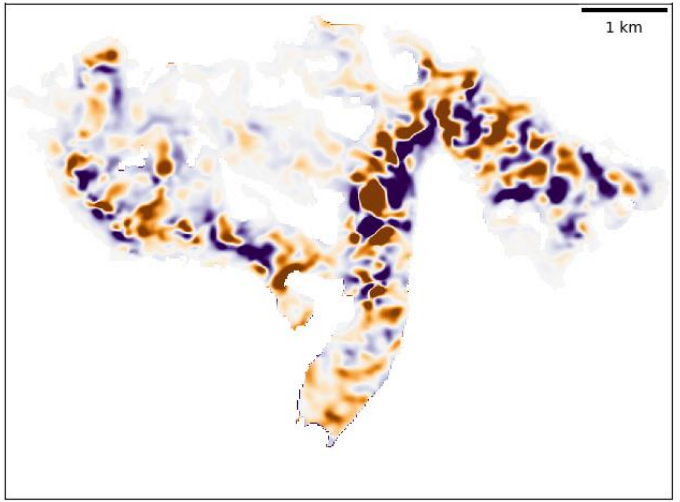


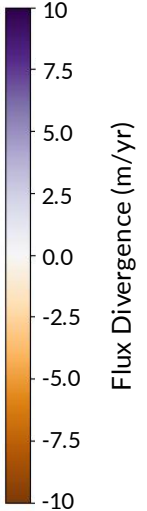






Flux Divergence (no smoothing)

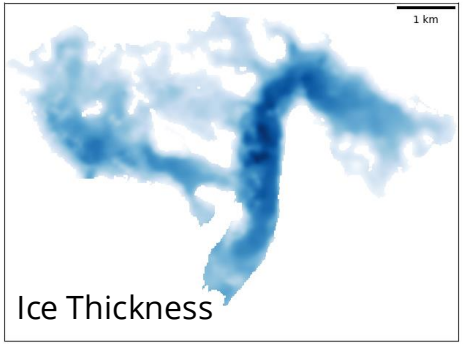


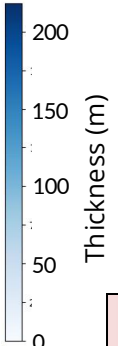


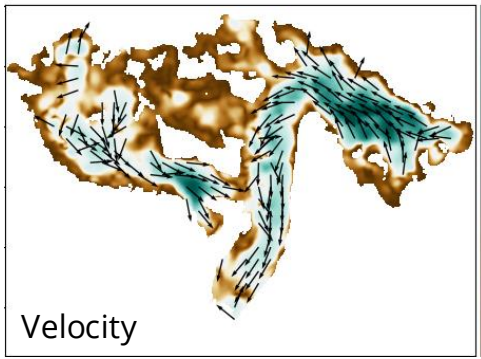
$$\nabla q = h \cdot \left(\frac{du_x}{dx} + \frac{du_y}{dy} \right) + u \cdot \left(\frac{dh}{dx} + \frac{dh}{dy} \right)$$

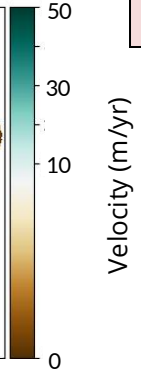
Millan et al. 2022, *Nat. Geosci.*

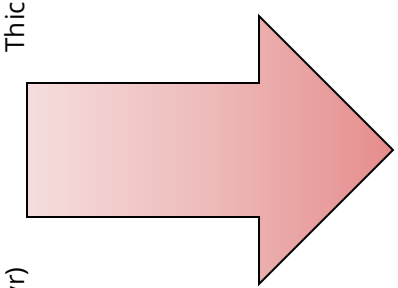
Remote sensing flux divergence is unreliable for point estimates



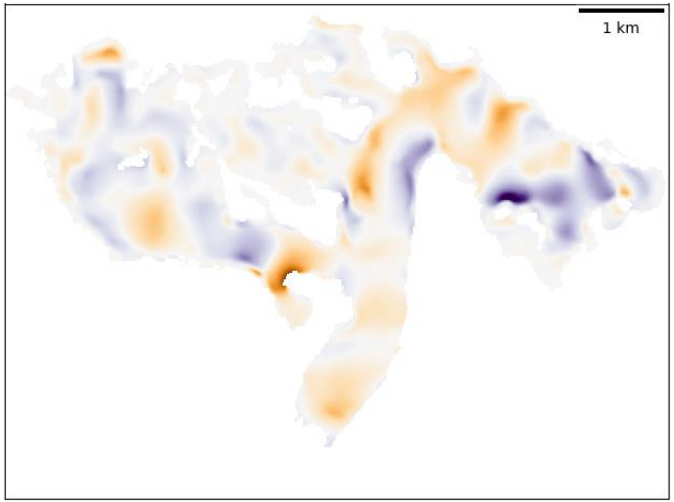


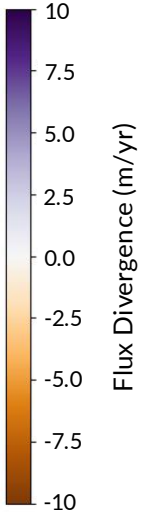






Flux Divergence (with smoothing)

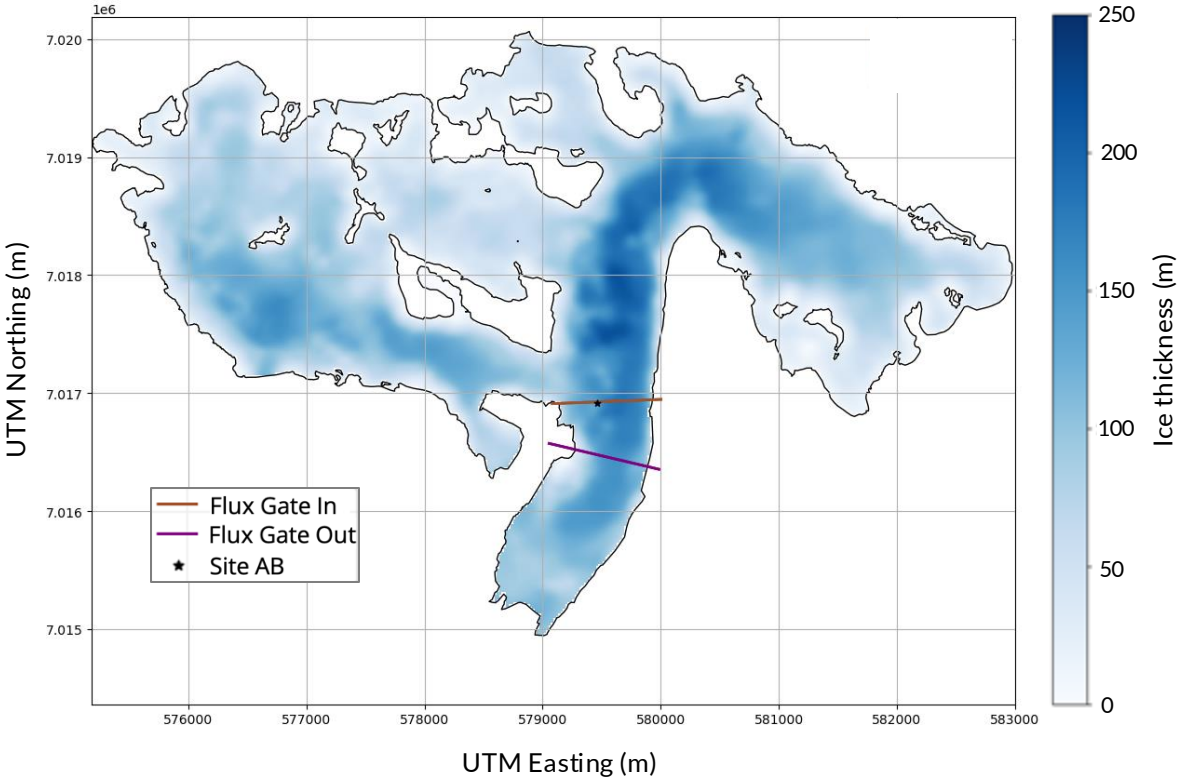




$$\nabla q = h \cdot \left(\frac{du_x}{dx} + \frac{du_y}{dy} \right) + u \cdot \left(\frac{dh}{dx} + \frac{dh}{dy} \right)$$

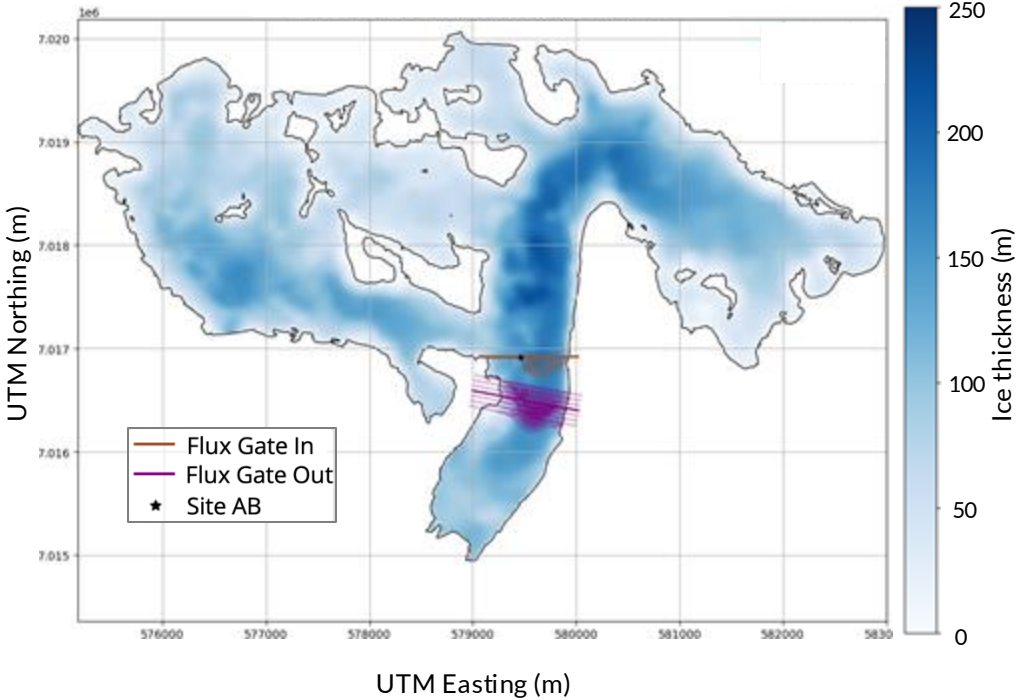
Millan et al. 2022, *Nat. Geosci.*

Flux gate approach can yield reasonable values



Parameter	Value
Flux Zone Area (sq.km):	0.34
Flux Zone DHDT (m/yr):	-1.70
Flux Zone DIVQ (m/yr):	1.89
Flux Zone CMB (m/yr):	-3.59

Flux gate approach is sensitive to gate placement

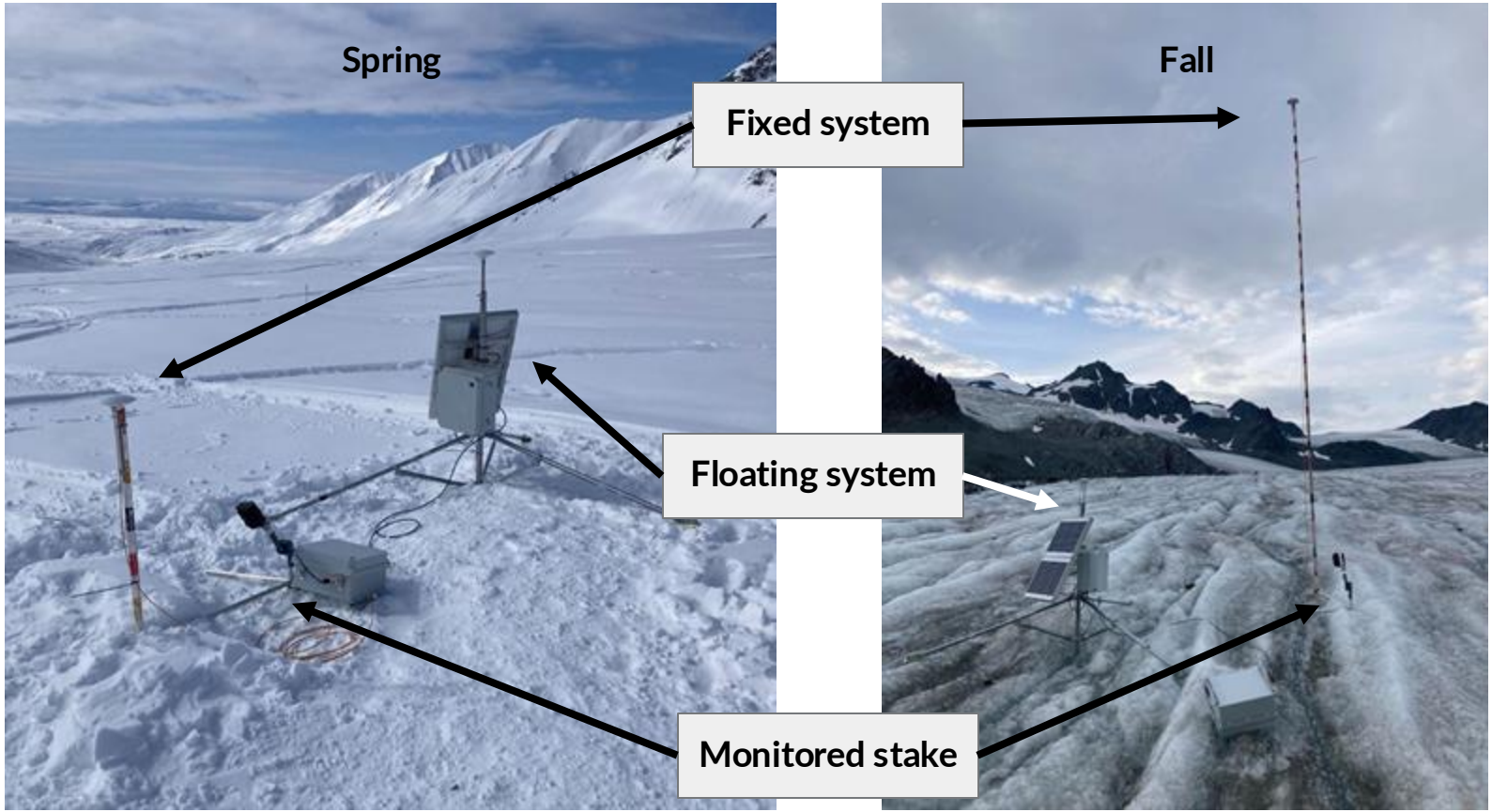


Parameter	Value	[min, max]
Flux Zone Area (sq.km):	0.31	[0.21, 0.41]
Flux Zone DHDT (m/yr):	-1.70	[-1.77, -1.67]
Flux Zone DIVQ (m/yr):	1.55	[0.32, 2.03]
Flux Zone CMB (m/yr):	-3.25	[-3.74, -2.01]

Can we use novel in-situ methods to get contemporaneous measurements of climatic mass balance, total mass balance, and flux divergence?

YES!

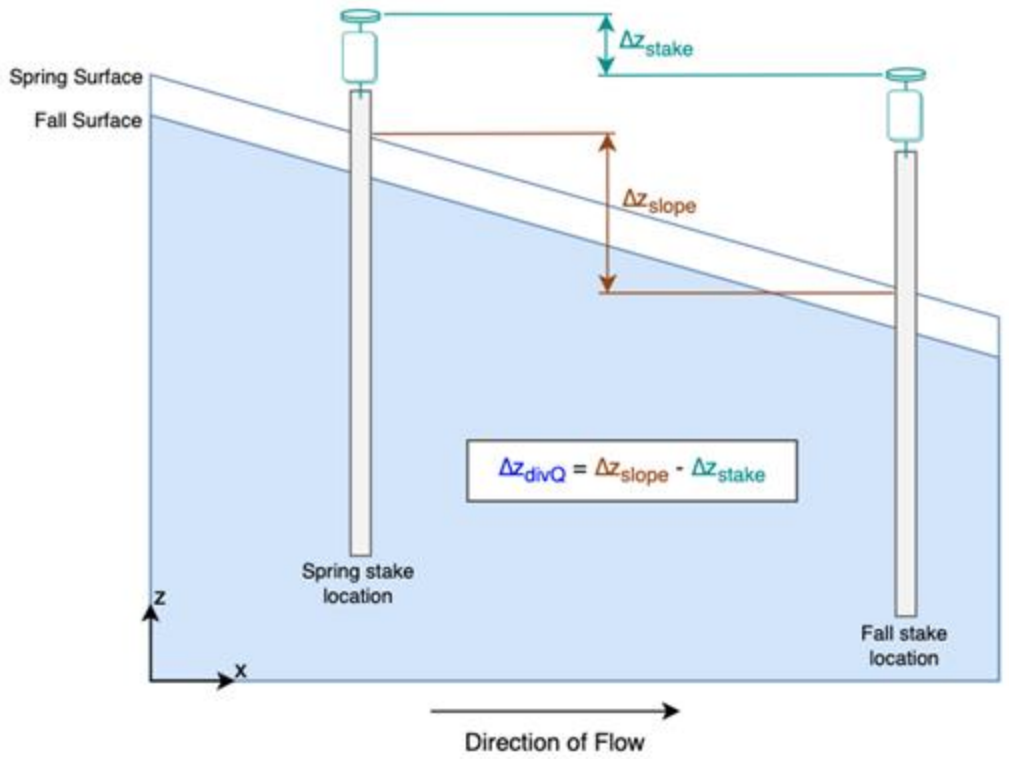
Cryologger GNSS Systems



Garbo, Cryologger Glacier Velocity Tracker (2023) GitHub

Instantaneous flux divergence

from GNSS station
'fixed' to ablation stake
(& DEM-derived slope)



Instantaneous flux divergence

from GNSS station
'fixed' to ablation stake
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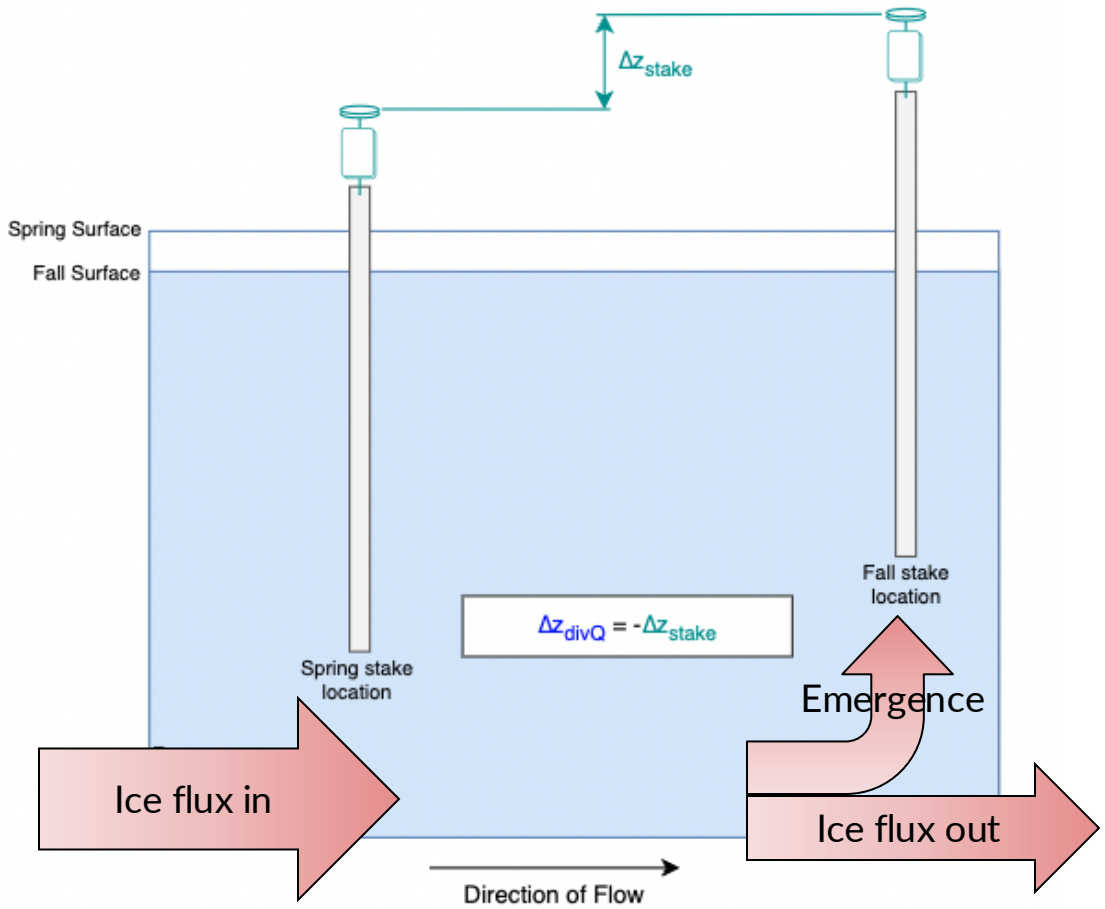
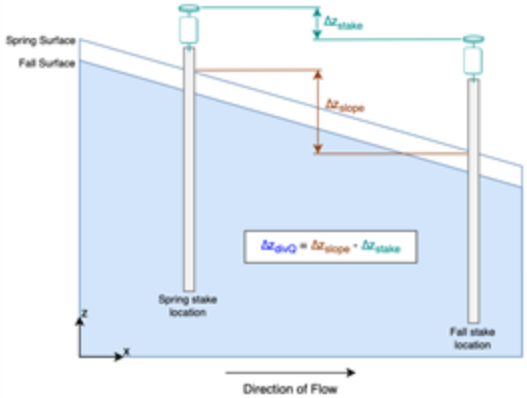
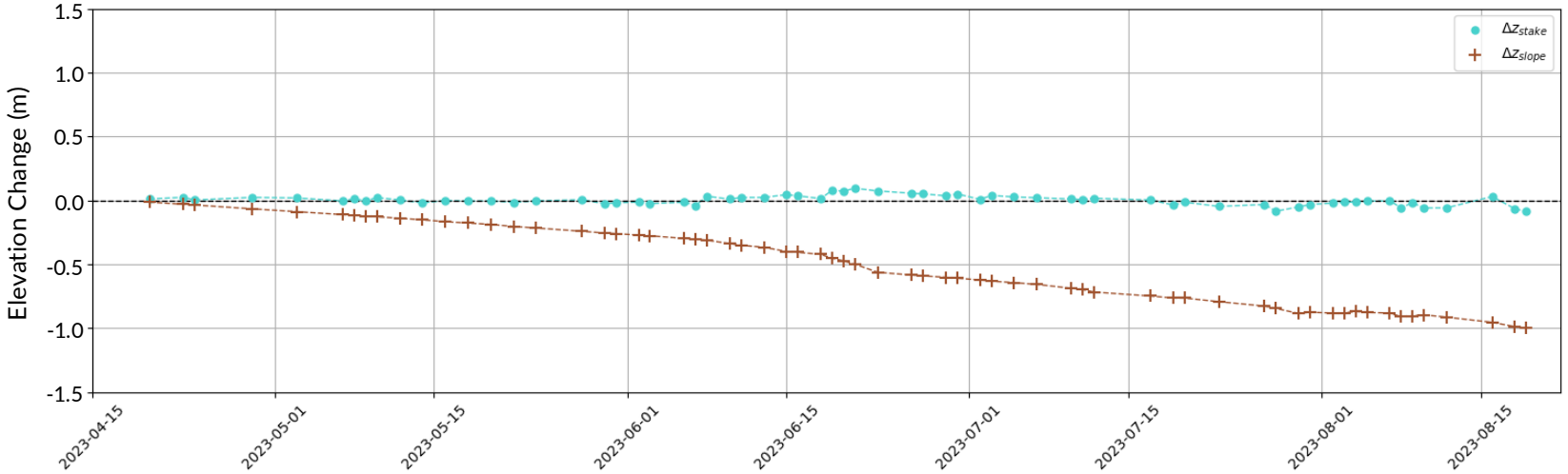


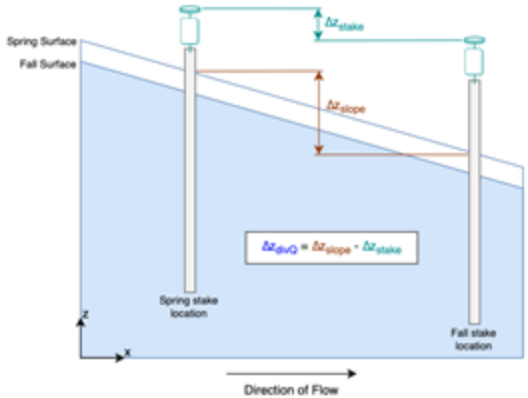
Results from Gulkana Site AB (ablation area)

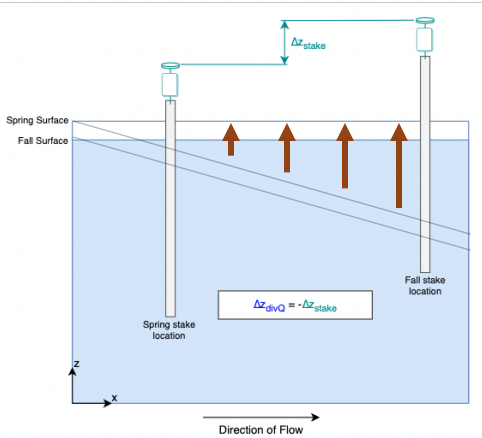


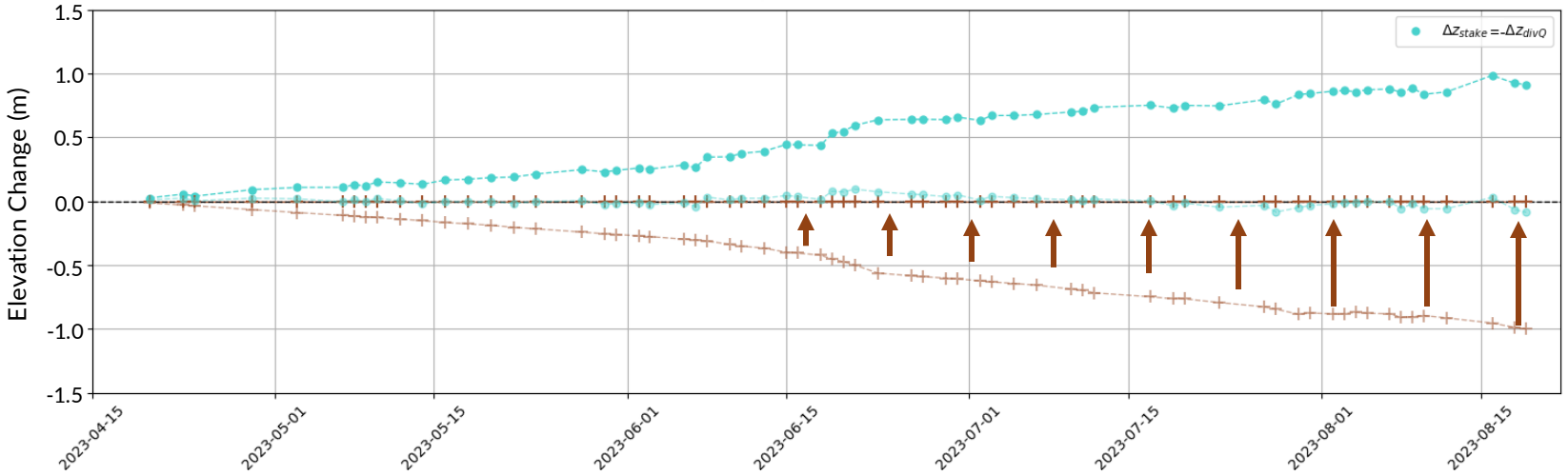
Gulkana Glacier Site AB Stake and Slope Elevation Change



Results from Gulkana Site AB (ablation area)



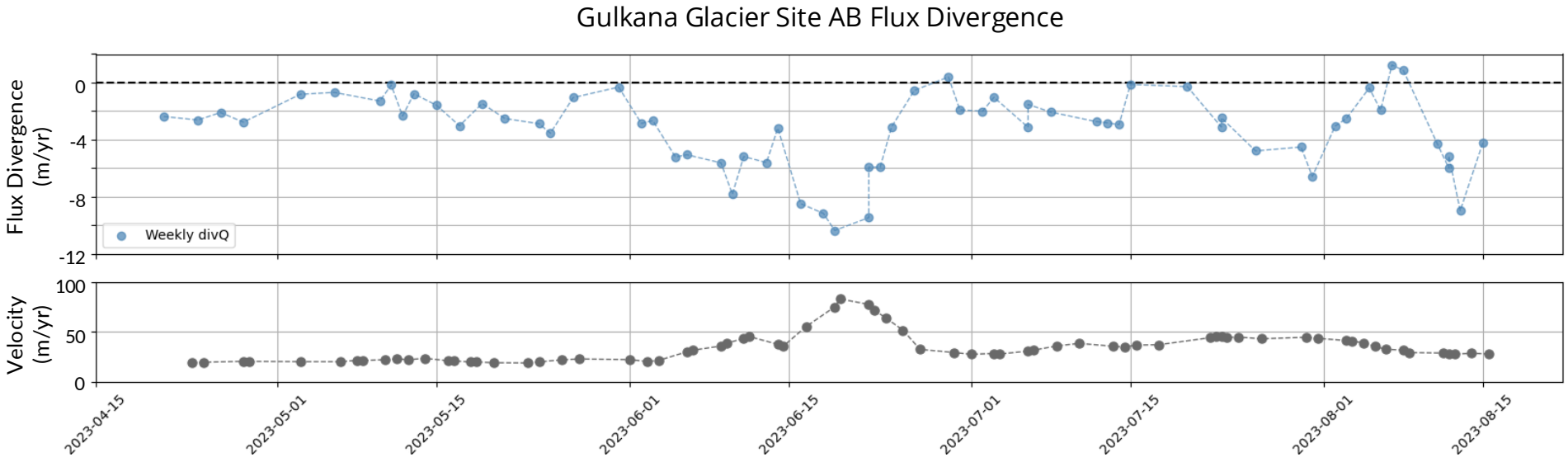




slope correction

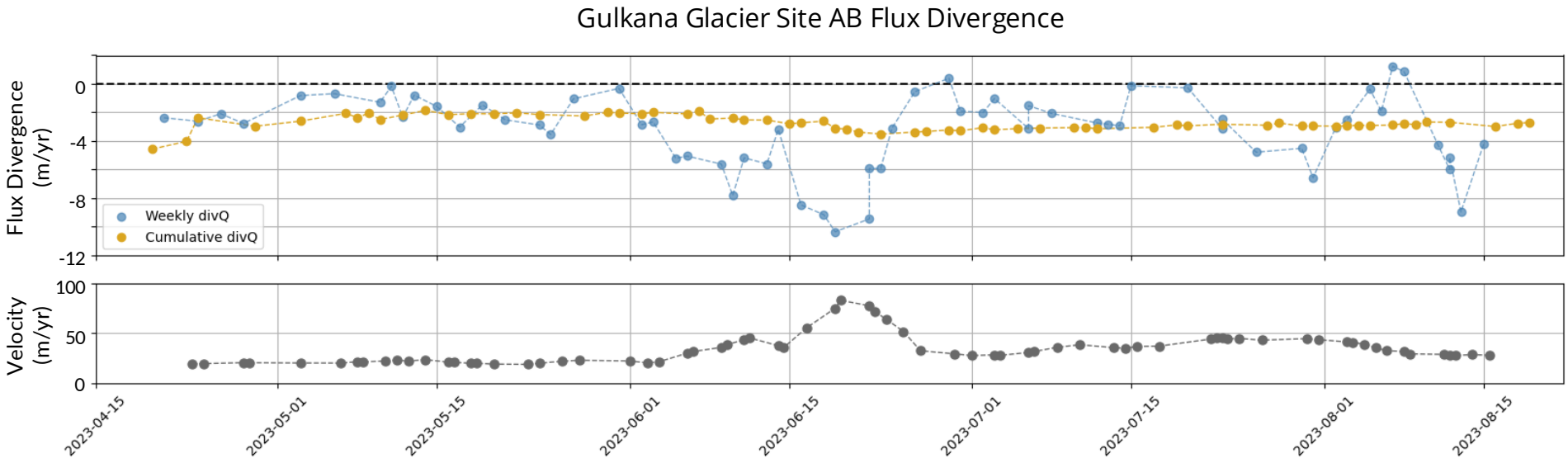
divide flux divergence by time for emergence velocity!

Weekly flux divergence reveals strong link with velocity

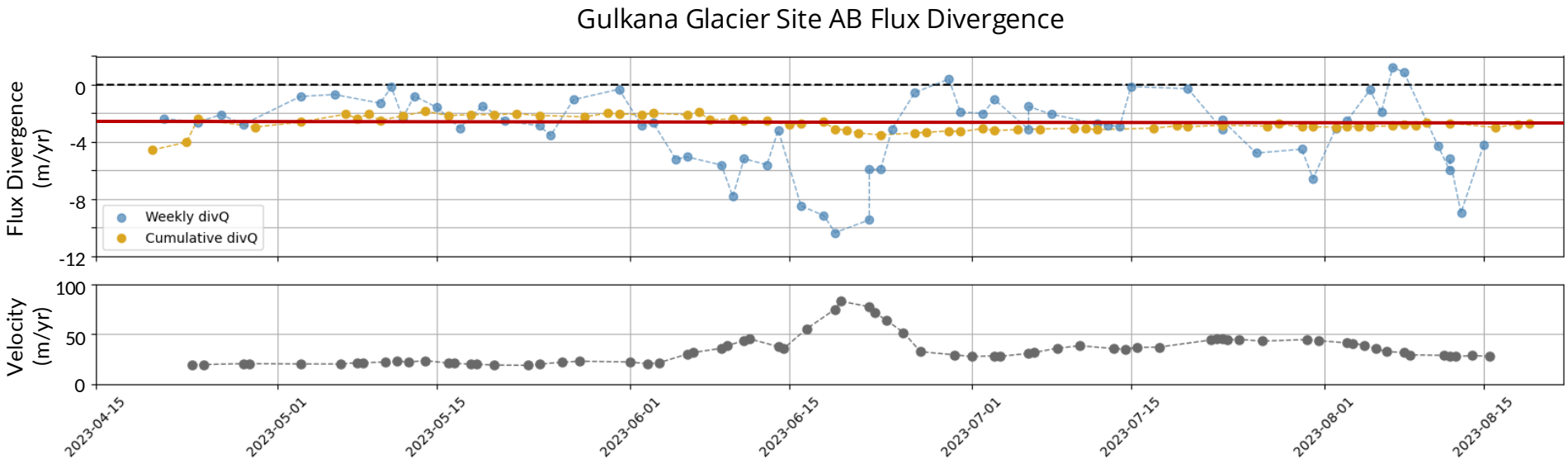


negative flux divergence is "emergence" *(following Cogley et al. 2011)*

Reliable flux divergence estimate after ~2 weeks



Reliable flux divergence estimate after ~2 weeks

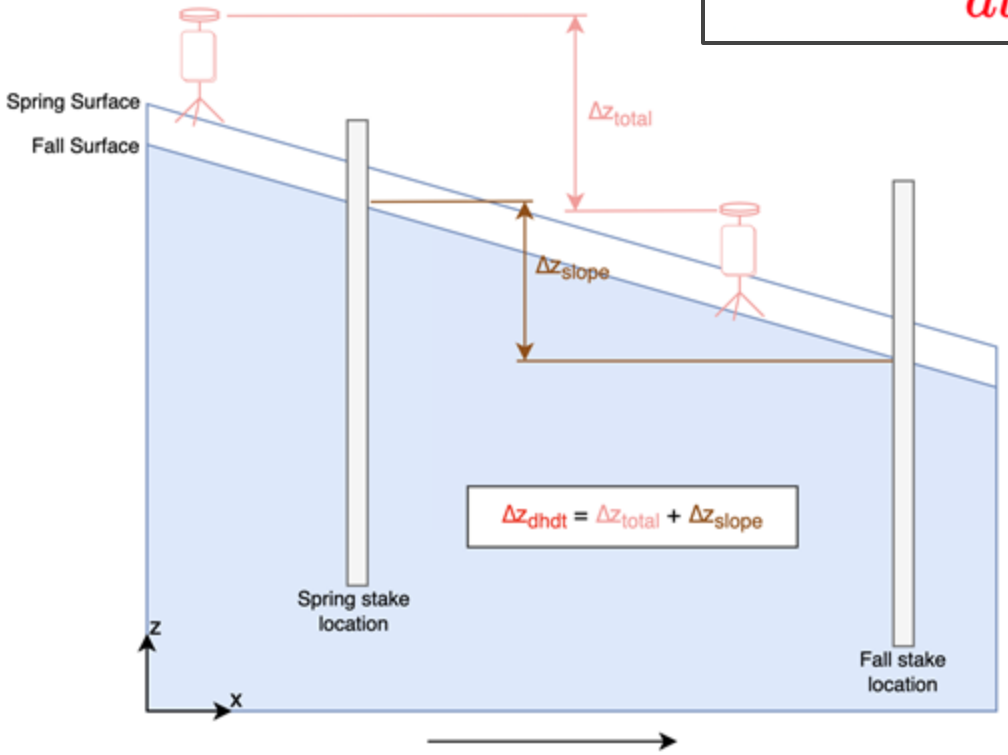


Spring-Fall divQ measurement: -2.6 m/yr

Instantaneous total mass balance

from GNSS station
'floating' on ice surface
(& DEM-derived slope)

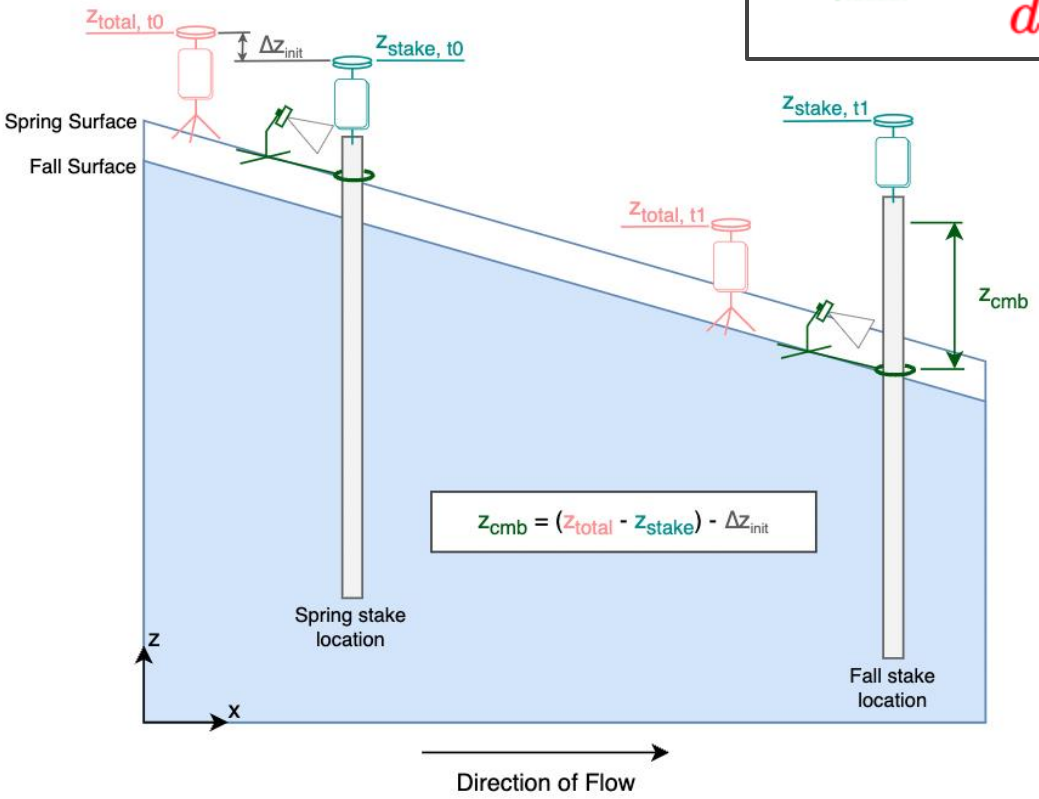
$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$



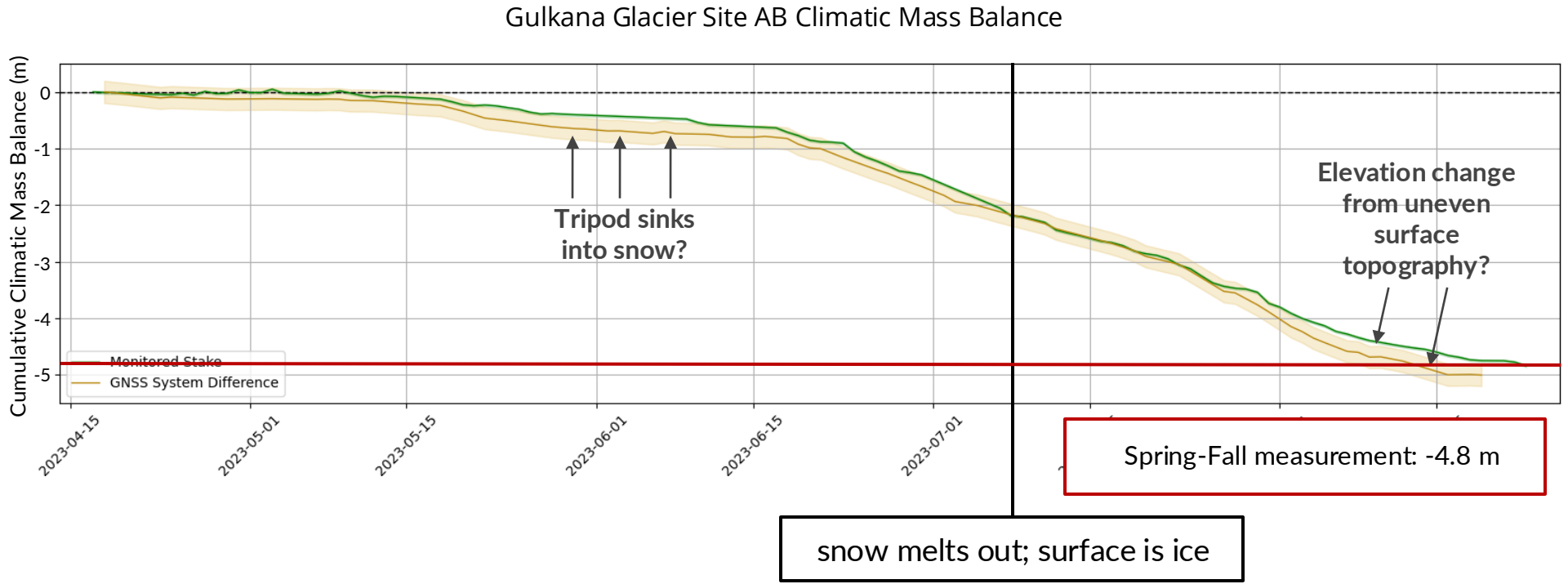
$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

Instantaneous climatic mass balance

from GNSS systems (with monitored ablation stake for validation)



Mass balance from GNSS systems



Takeaways

- Potential for “robust” methods of flux divergence from remote sensing
  - Require ground-truth measurements

- GNSS system fixed to an ablation stake enables precise flux divergence
- ***A fixed GNSS system with a monitored ablation stake accurately derives all elements of the continuity equation***
- The fixed-floating system has redundancy for flux divergence and climatic mass balance

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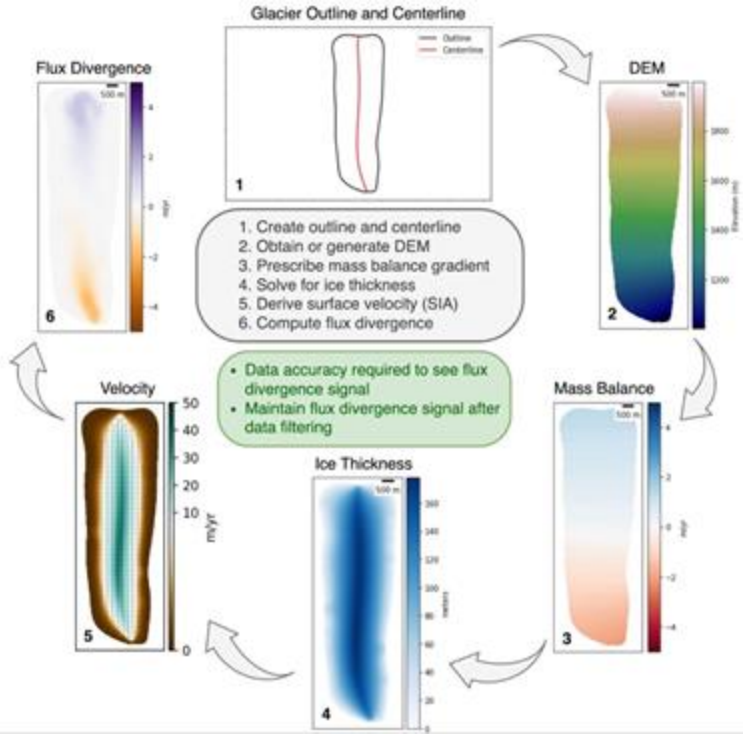
Next Steps

- Deployment of more systems next Spring
- GNSS-IR?

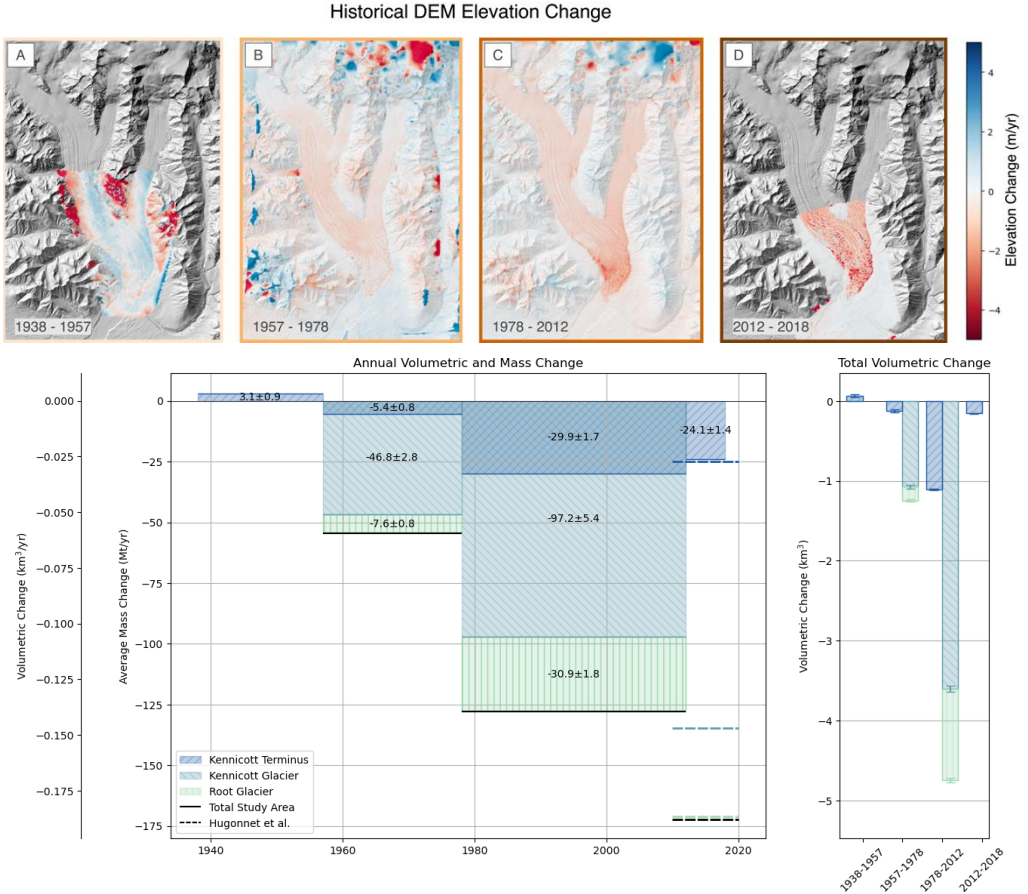
…other projects I'm excited about…

Theoretical flux divergence approach



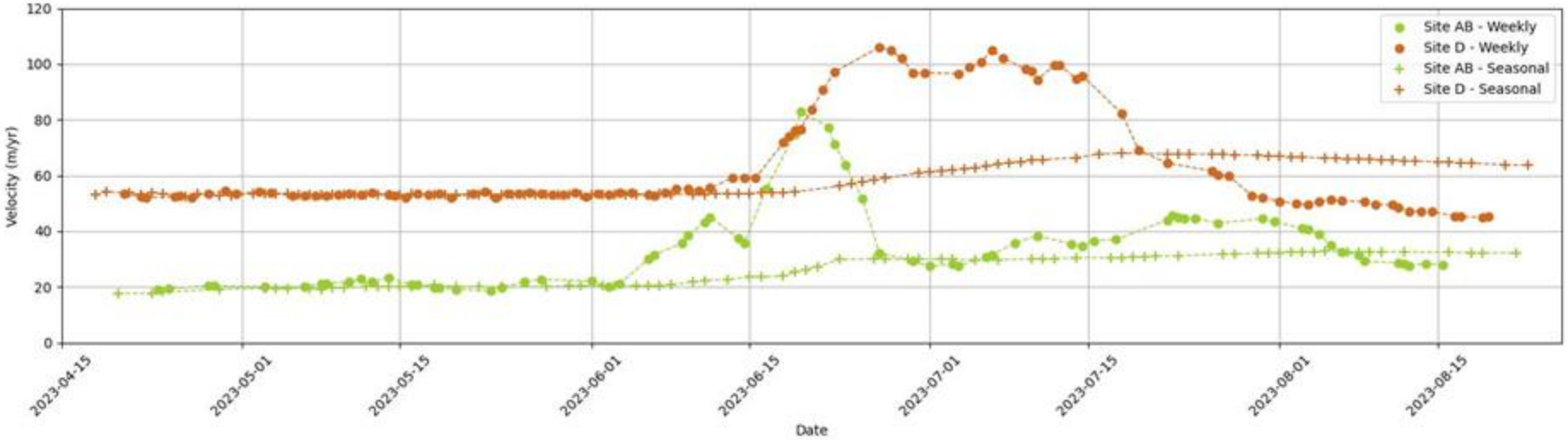
- simplify and manipulate geometries
- control for noise/bias in data
- assess error and uncertainty propagate through velocity and ice thickness

Long-term changes on Kennicott from historical imagery



Thank you! Questions?

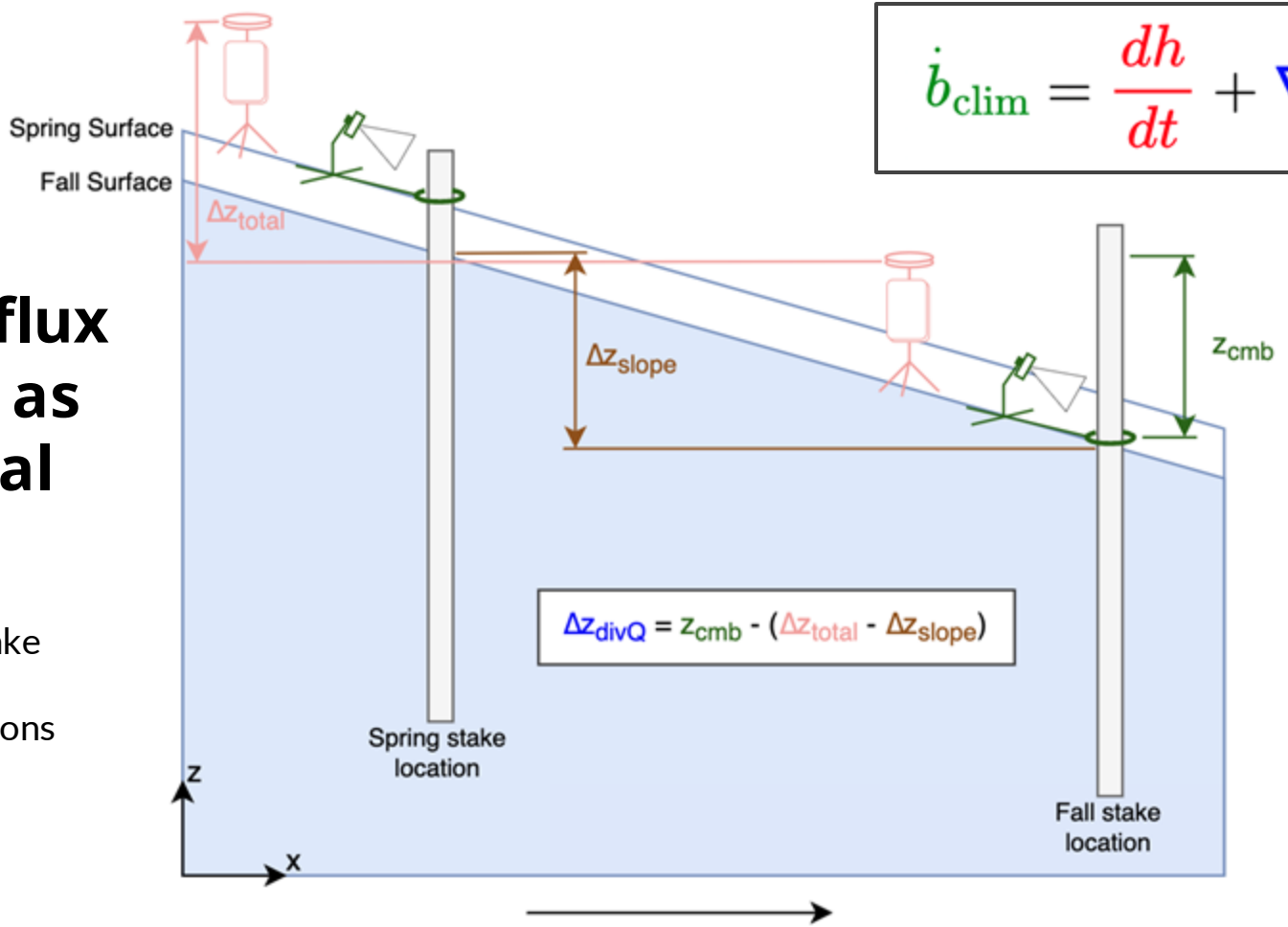
Velocity Signals at Sites AB (terminus) and D (accumulation area)



Calculating flux divergence as the residual

from monitored stake with ‘fixed’ and ‘floating’ GNSS stations

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

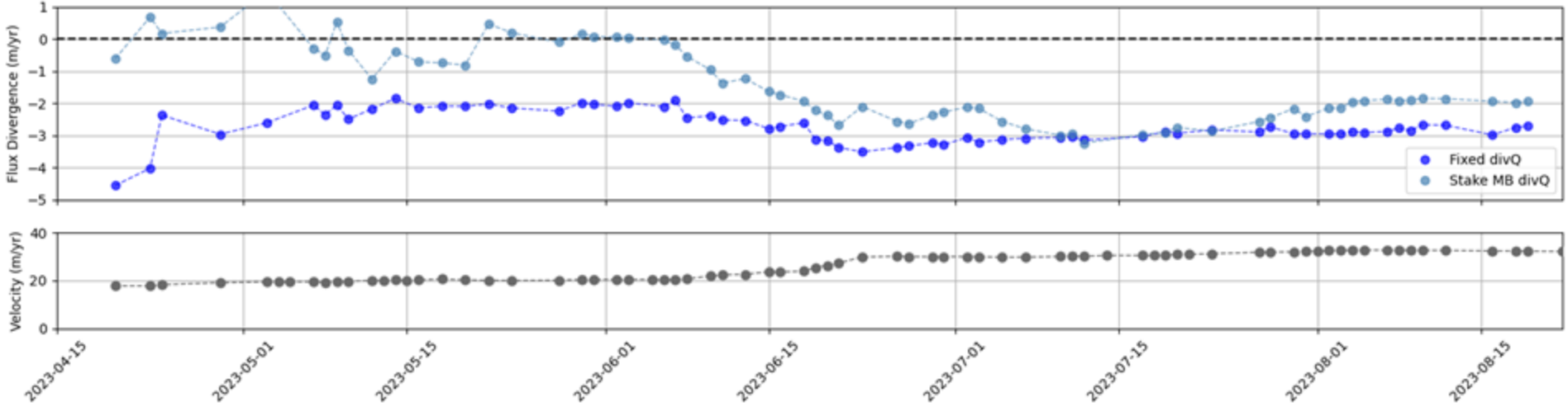


Independent flux divergence methods show decent agreement

$$\Delta z_{divQ} = \Delta z_{slope} - \Delta z_{stake}$$

$$\Delta z_{divQ} = z_{cmb} - (\Delta z_{total} - \Delta z_{slope})$$

Gulkana Glacier Site AB ***Cumulative*** Flux Divergence Comparison

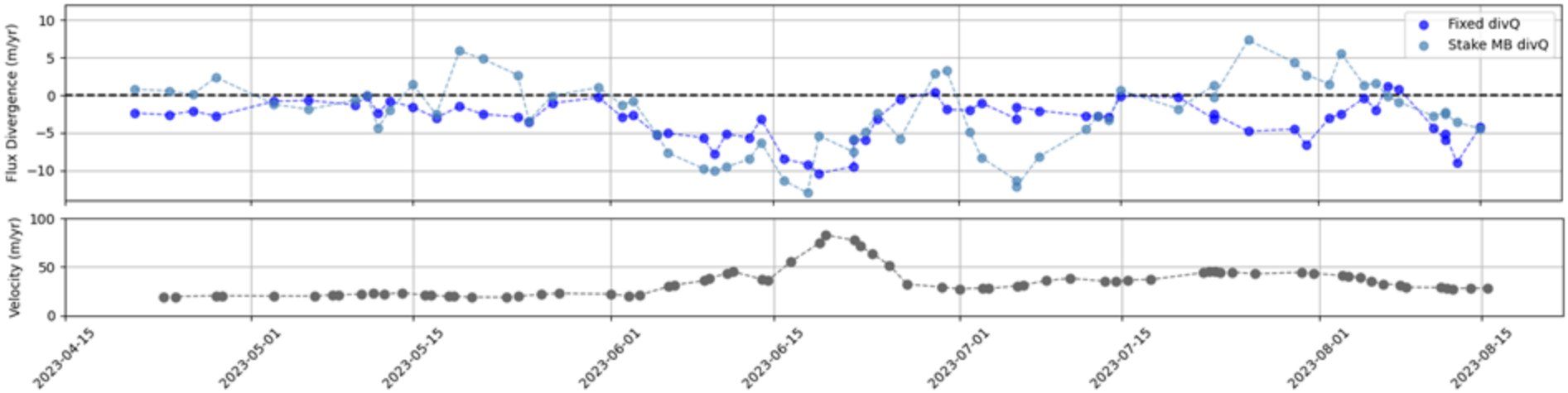


Both methods see the same flux divergence speed-up

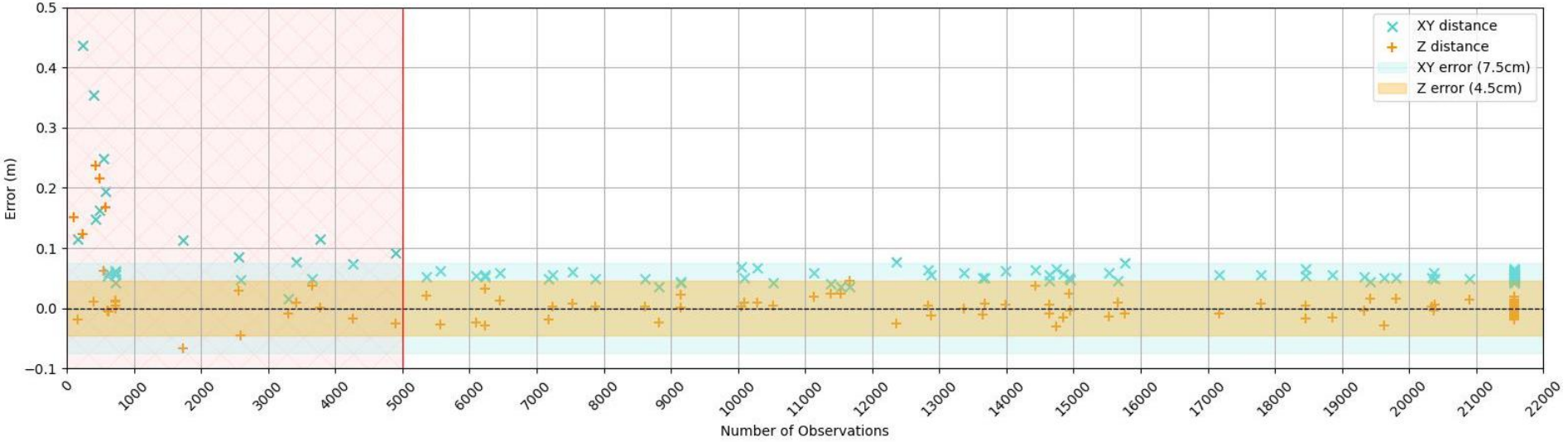
$$\Delta z_{divQ} = \Delta z_{slope} - \Delta z_{stake}$$

$$\Delta z_{divQ} = z_{cmb} - (\Delta z_{total} - \Delta z_{slope})$$

Gulkana Glacier Site AB ***Weekly*** Flux Divergence Comparison



GNSS System Accuracy – Base Station results



Full Overview

Climatic Mass Balance (2 methods):

from monitored stake

$z_{cmb} = z_{total} - z_{stake}$

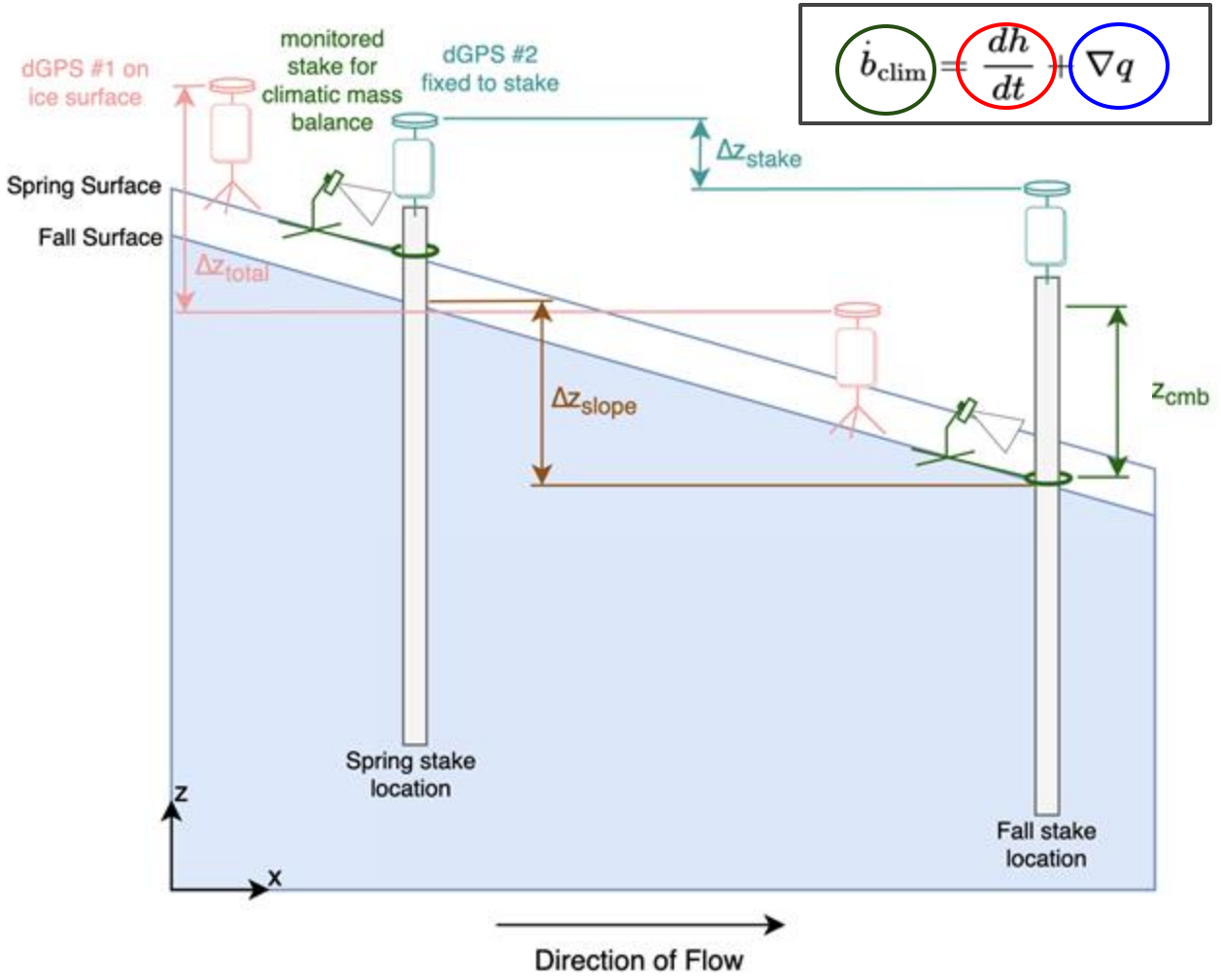
Total Mass Balance:

$\Delta z_{dhdt} = \Delta z_{total} - \Delta z_{slope}$

Flux Divergence (2 methods):

$\Delta z_{divQ} = \Delta z_{slope} - \Delta z_{stake}$

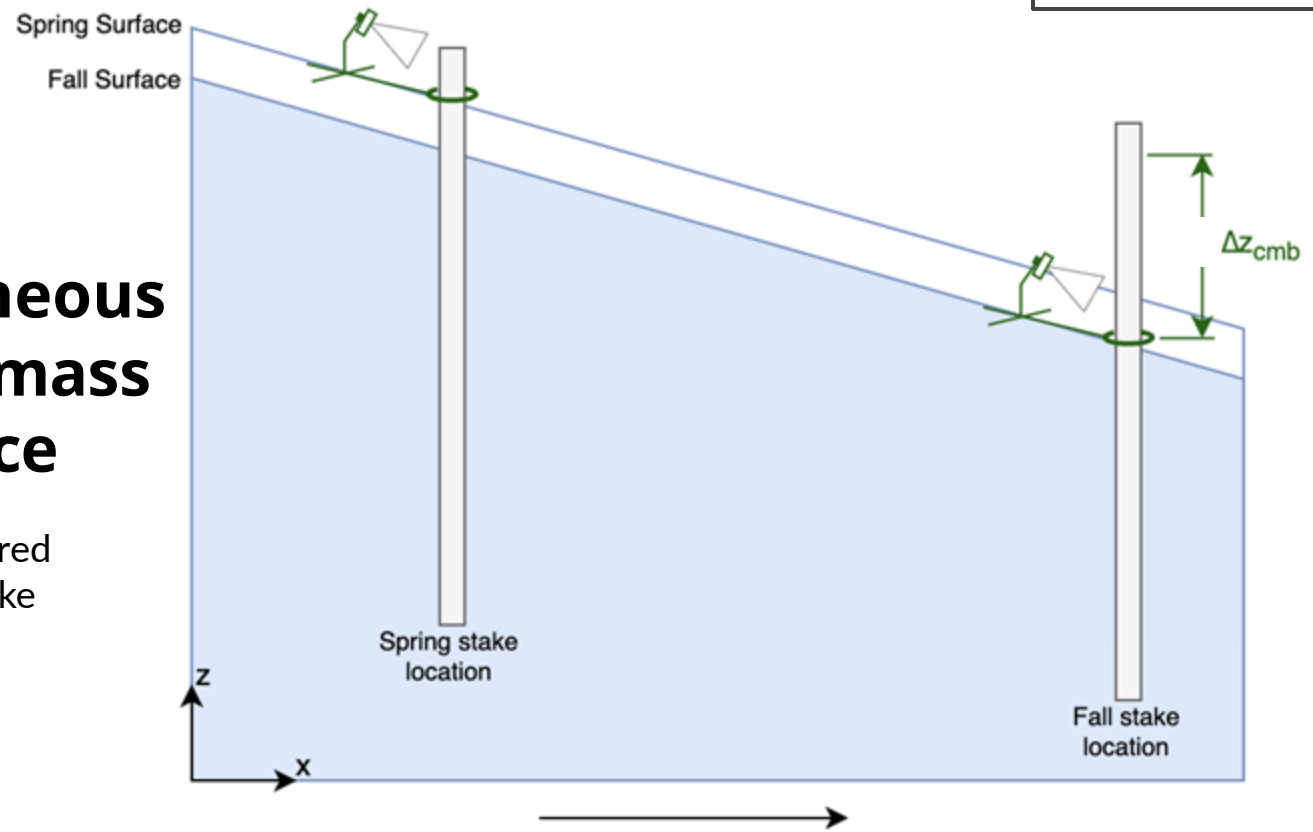
$\Delta z_{divQ} = z_{cmb} - (\Delta z_{total} - \Delta z_{slope})$



$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

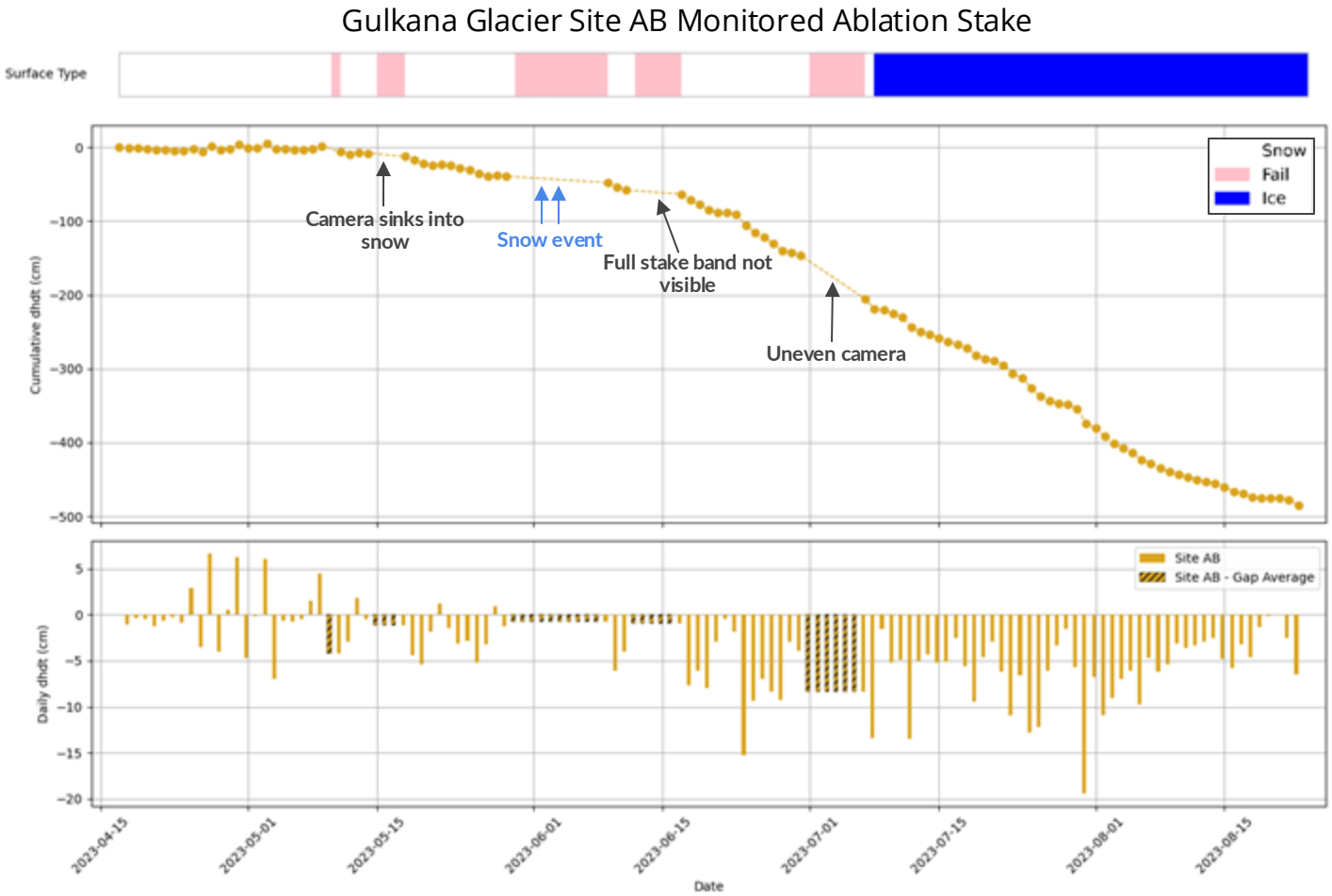
Instantaneous climatic mass balance

from monitored ablation stake

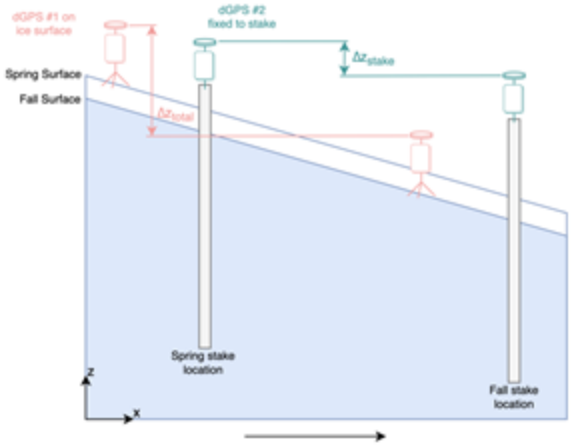


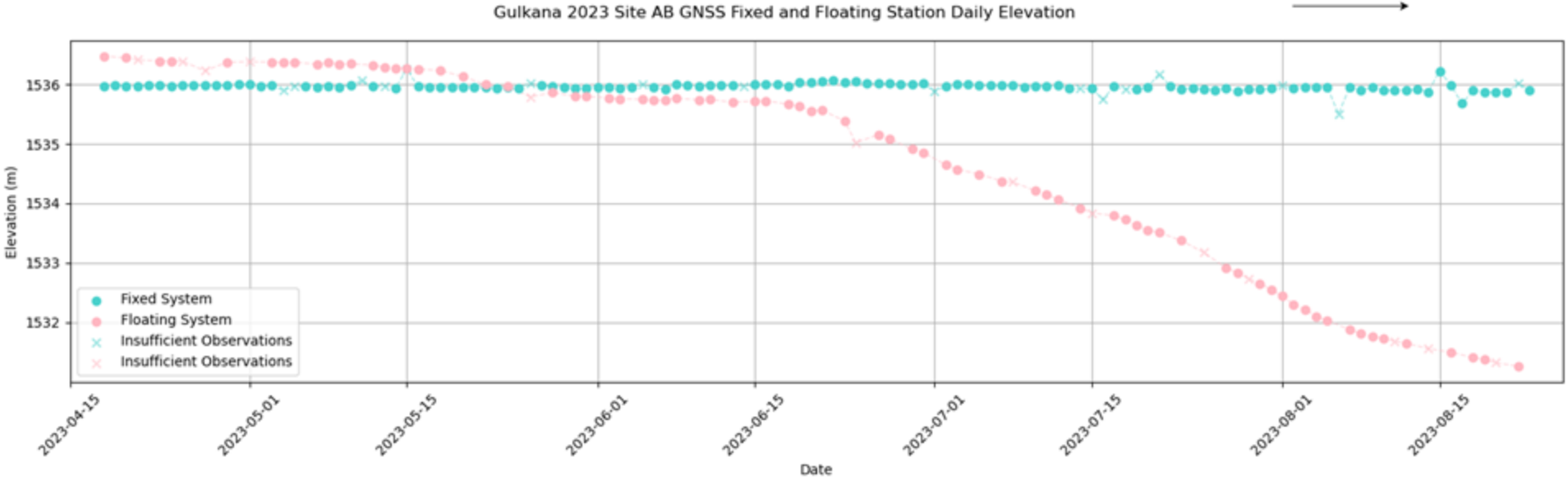
Monitored Ablation Stakes

climatic mass balance record

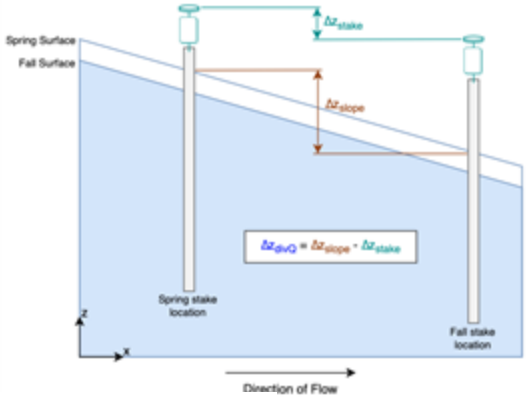


Site AB GNSS Elevation Results

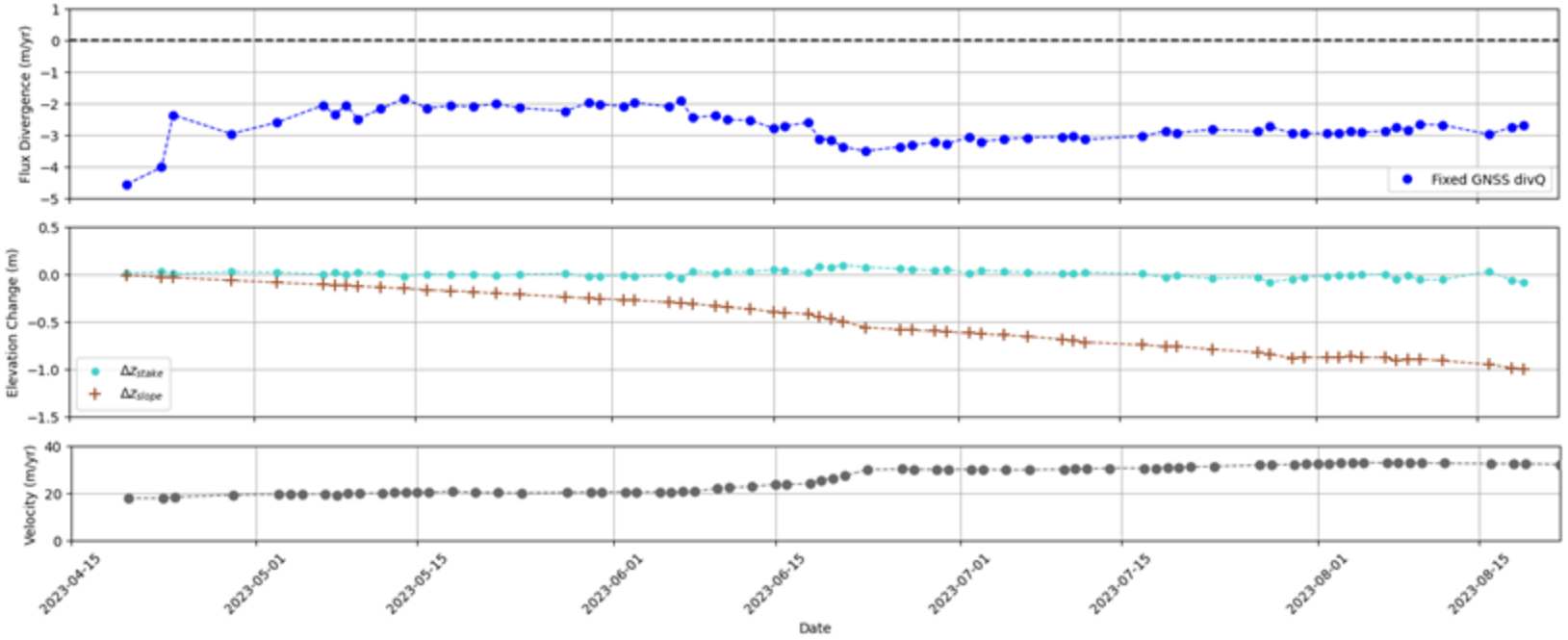




Reliable flux divergence estimate after ~2 weeks

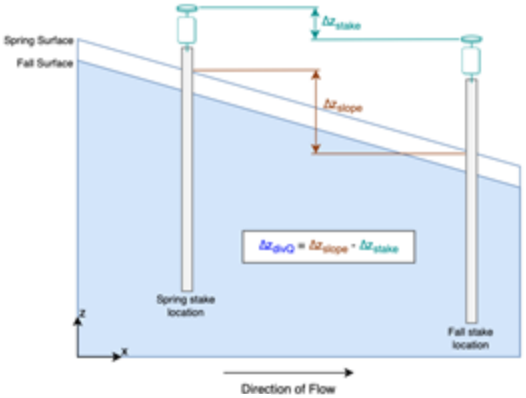


Gulkana Glacier Site AB ***Cumulative*** Flux Divergence

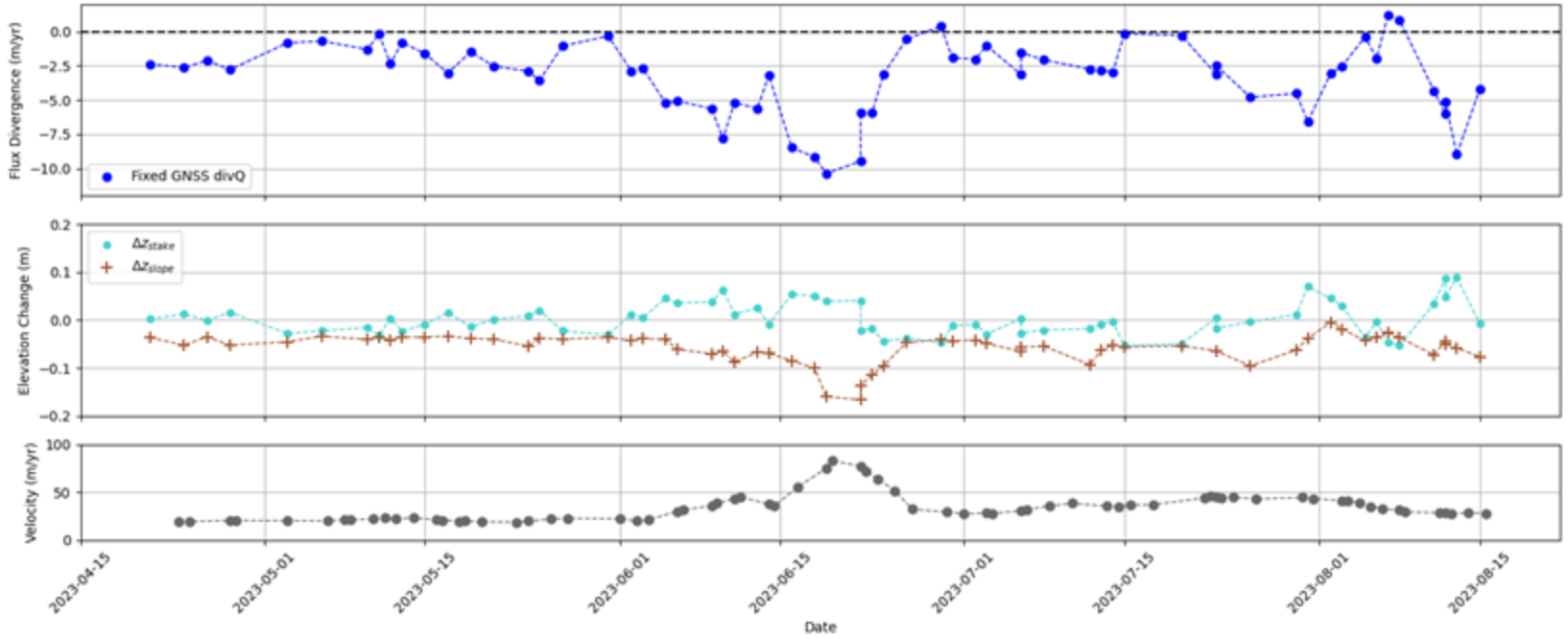


negative flux divergence is emergence; this is just a sign convention

Weekly flux divergence reveals strong link with velocity



Gulkana Glacier Site AB ***Weekly*** Flux Divergence

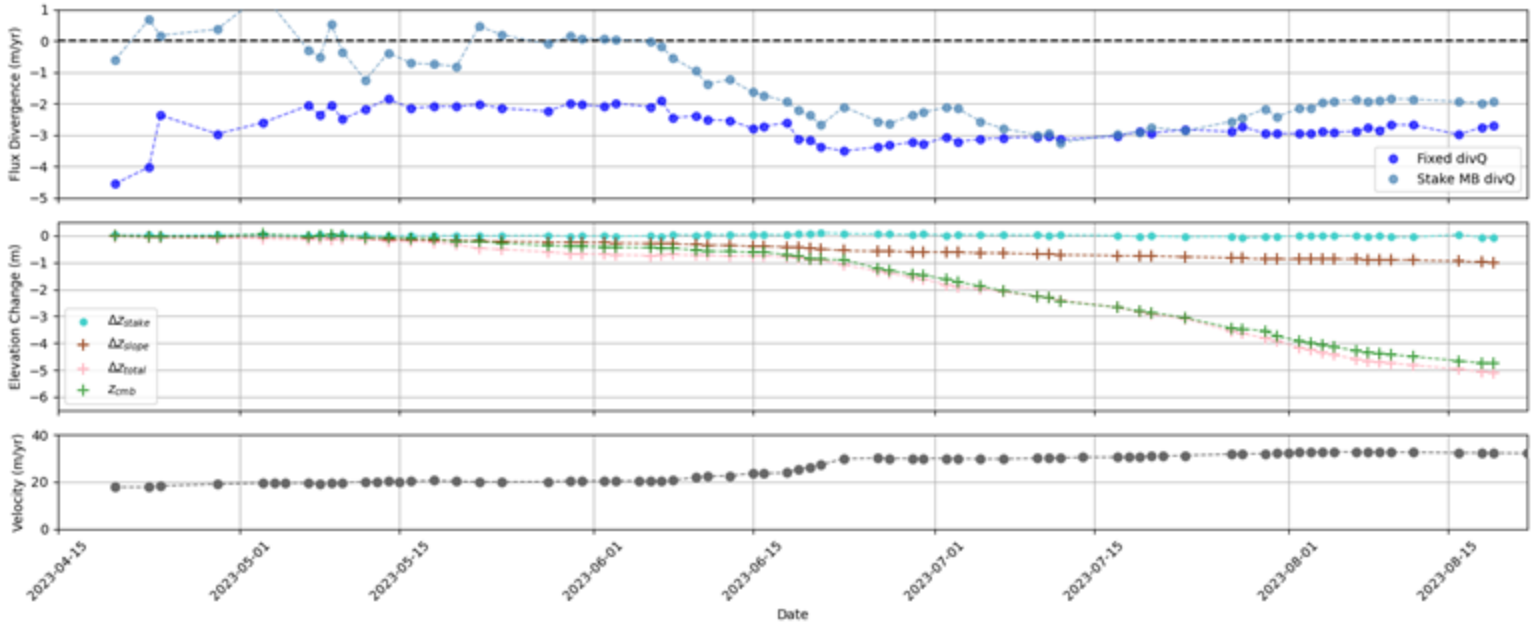


Instantaneous flux divergence

$$\Delta z_{divQ} = \Delta z_{slope} - \Delta z_{stake}$$

$$\Delta z_{divQ} = z_{cmb} - (\Delta z_{total} - \Delta z_{slope})$$

Gulkana Glacier Site AB ***Cumulative*** Flux Divergence Comparison

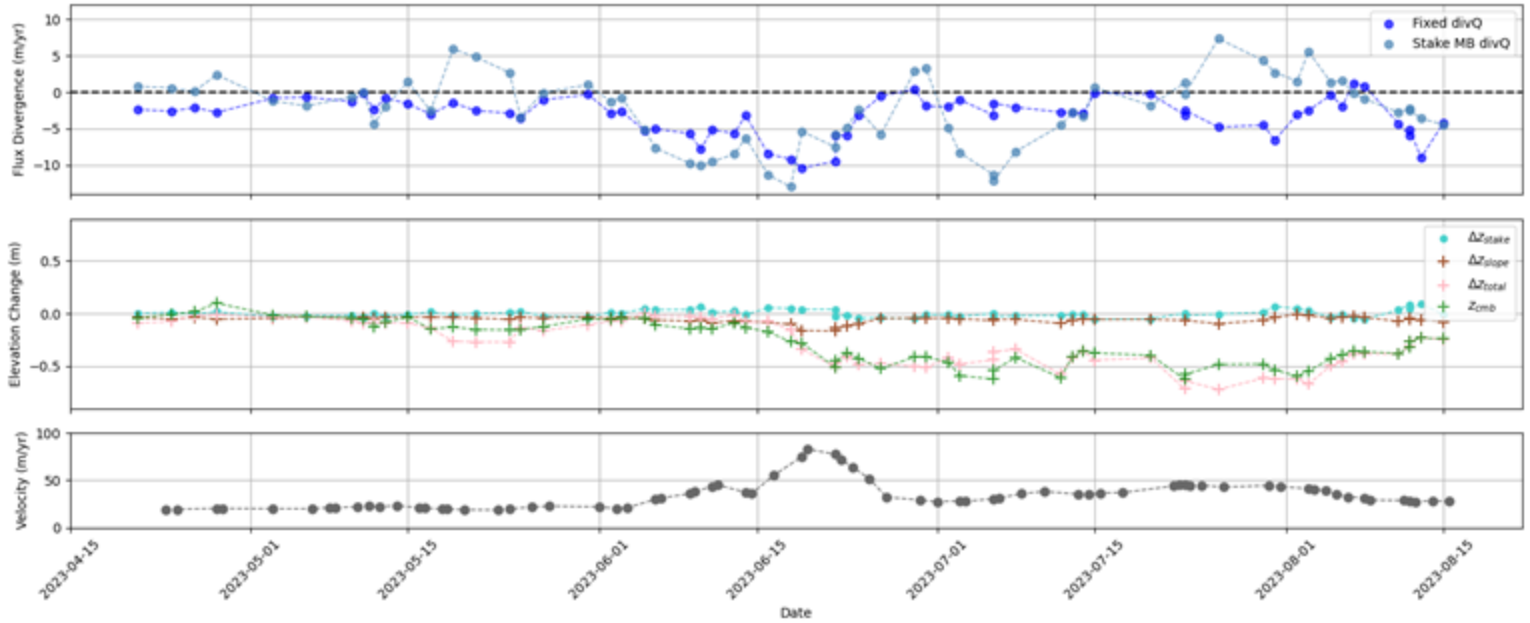


Instantaneous flux divergence

$$\Delta z_{divQ} = \Delta z_{slope} - \Delta z_{stake}$$

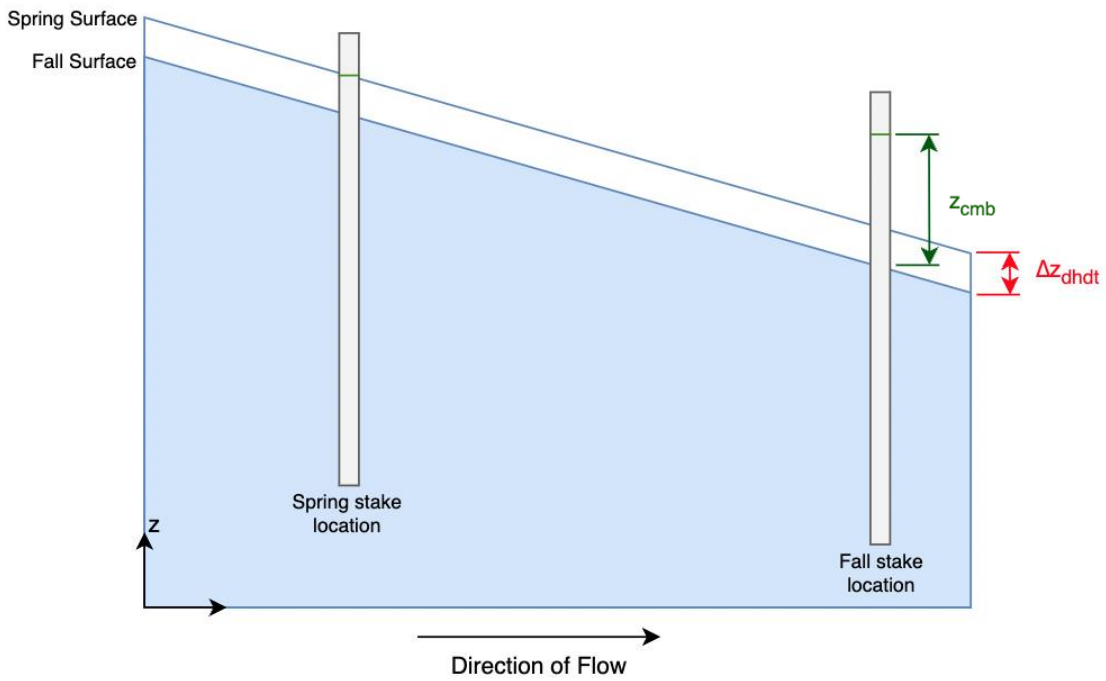
$$\Delta z_{divQ} = z_{cmb} - (\Delta z_{total} - \Delta z_{slope})$$

Gulkana Glacier Site AB ***Weekly*** Flux Divergence Comparison



Flux divergence from ablation stakes

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$



Ablation stakes give us summer flux divergence

$$\dot{b}_{\text{clim}} = \frac{dh}{dt} + \nabla q$$

Gulkana Site AB -- Summer ’23:

From ablation stake measurements:

- Climatic Mass Balance: -4.85 m
- Total Mass Balance: -3.93 m

- **Flux Divergence: -0.92 m**
  - ~-2.62 m/yr