Characterization of Silicone Polymers for Energy Harvesting from Compliant Membrane Foils

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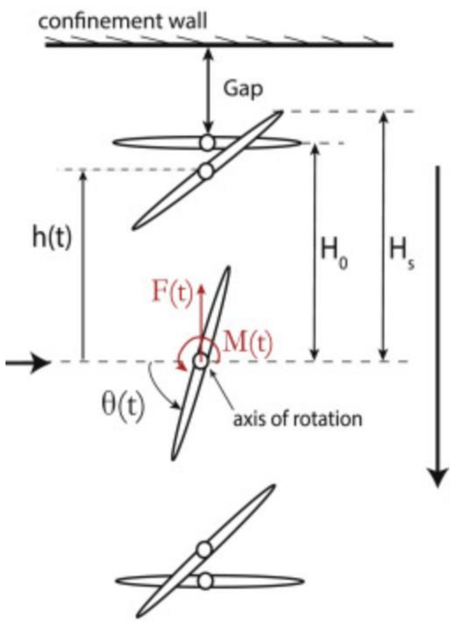
Background

- Hydrokinetic energy extraction from hydrodynamic foils has shown a lot of promise as a minimally invasive renewable energy source from tidal and riverine flows
- Foils struggle to match the efficiency of standard rotary turbines
- Passive, shape-morphing 'compliant' foils can be used to boost efficiency
  - Membrane material in foils camber and interact with water flow, which stabilizes LEVs and increases lift forces that drive the foil
  - Silicone polymer material is synthesized and cured from liquid polymer base, a diluted cross-linker, and a thinning agent
    - Amount of thinning agent is adjusted for desired membrane elasticity/stiffness



Su et al., 2019

Silicone Polymers: Overview and Synthesization

- Uncured liquid silicone undergoes a platinum-based addition curing reaction called hydrosilylation, in which a polymer base is mixed with a diluted crosslinker.
  - 4 parts: Part A, Part B ‘Fast’, Part B ‘Slow’, Part C
  - Each part is stable and unreactive by itself

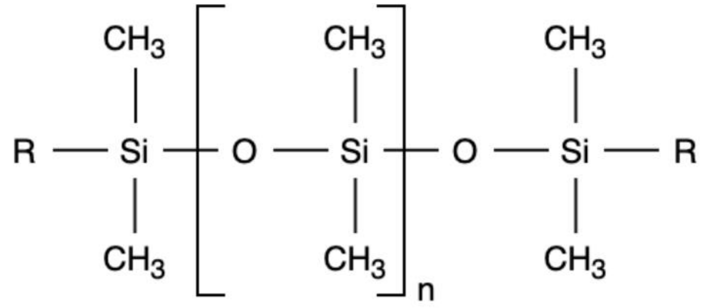


Figure 3.1. Silicone polymer molecular structure

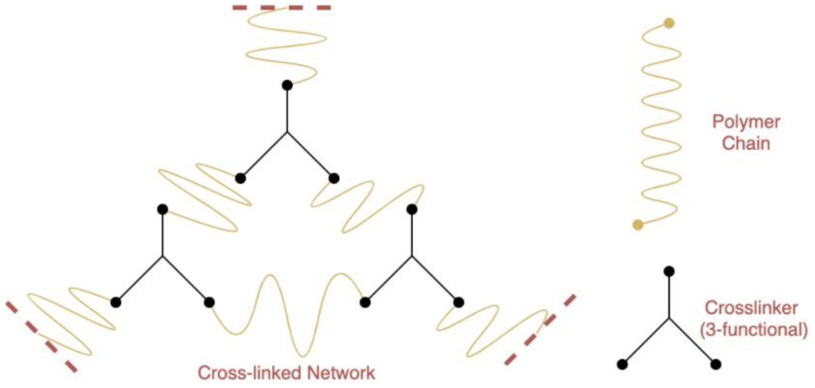


Figure 3.2. Ideally linked silicone network structure

Compliant Membrane Hydrofoils

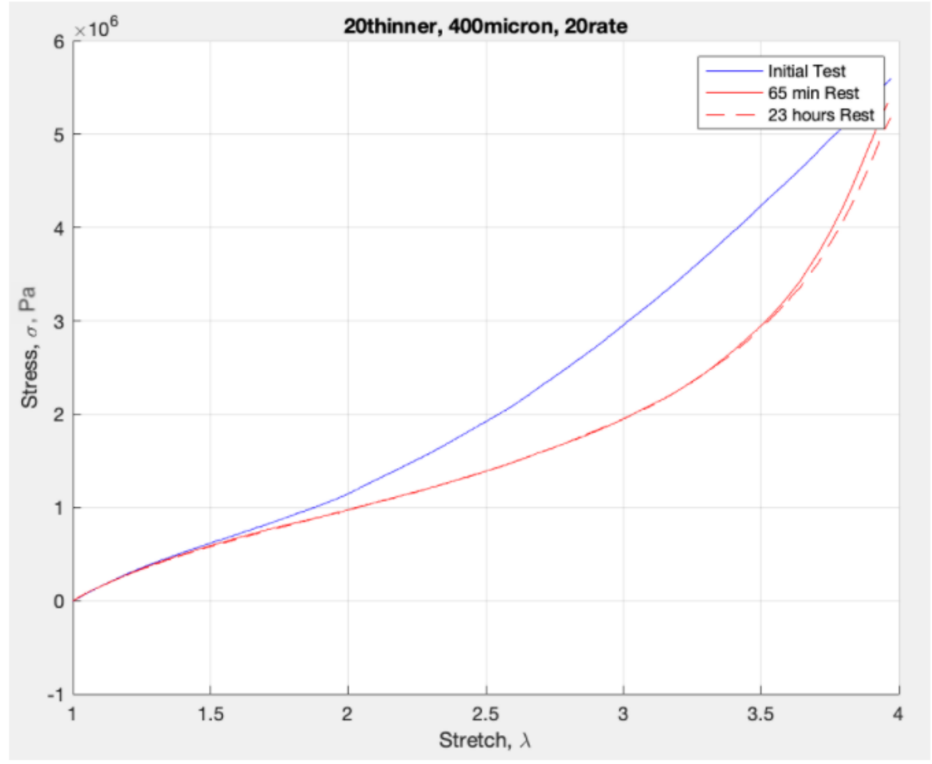
- We adjust thickness and total weight percentage of Part C (thinner) for desired material stiffness
  - Thickness typically ranges from around 300-500 microns
  - Part C ranges from 5-50% of total silicone mass
- Also prescribe a pre-stretch in foils, usually either 5-10%

My Project and Goals

- Characterize the silicone polymer materials used for energy harvesting so behavior is understood and known
  - Series of uniaxial tests and nonlinear hyperelastic model fitting
  - Ring-down analysis of mechanical oscillator to estimate damping
- Investigate the potential of a mechanical oscillator to estimate material properties
  - Low cost, low tech alternative to uniaxial machines
  - Test at high strain rates to try to bring out viscoelastic behavior

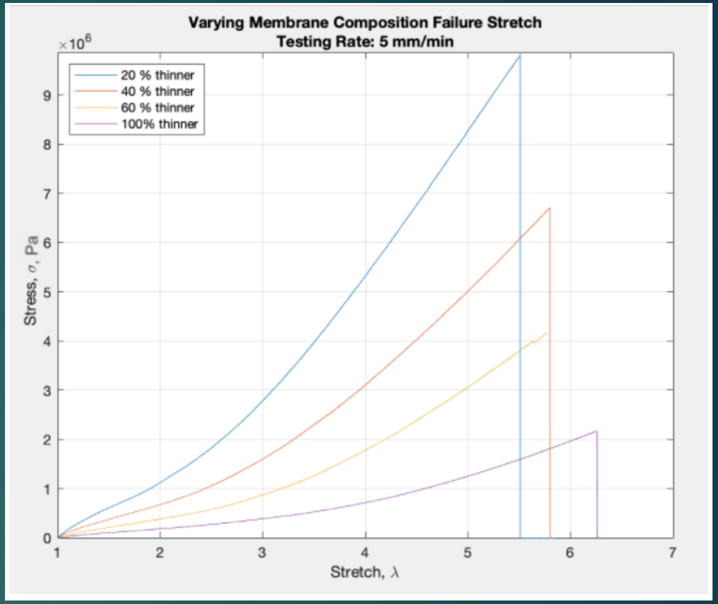
Evidence of Viscoelastic Effects

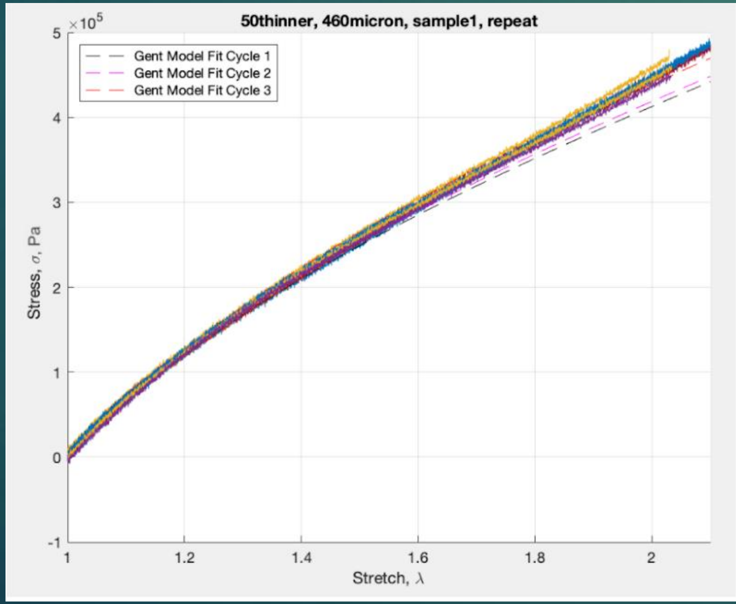
- Mullins Effect evident in samples stretched beyond any previous maximum stretching
- Permanent set is also evident as samples do not return exactly to their original length
- Need to establish a procedure for uniform testing to eliminate this bias in some samples

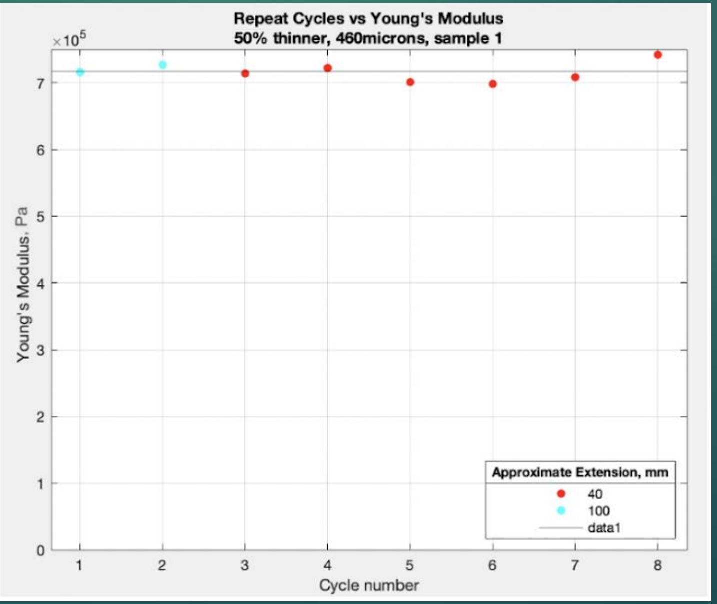


Developing Testing Method

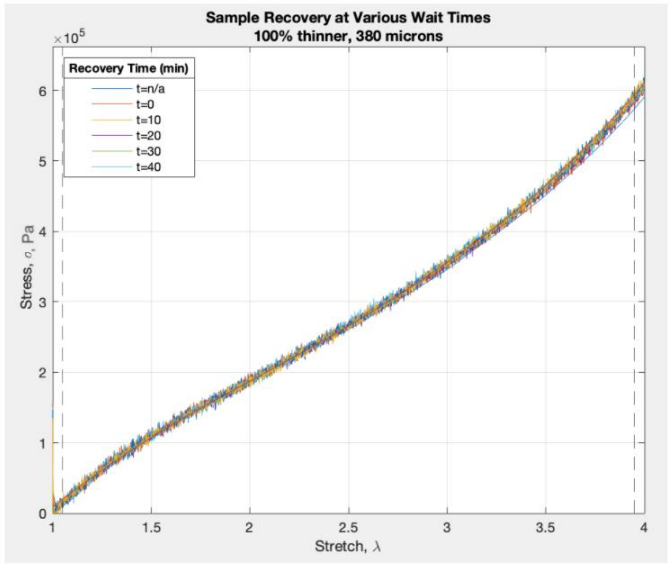
- Apply manual pre-stretch after laser-cutting samples to eliminate Mullins effect
- Define appropriate stretch range to avoid permanent deformation
- Analyze material behavior over longer periods of time
- Establish wait time for sample testing

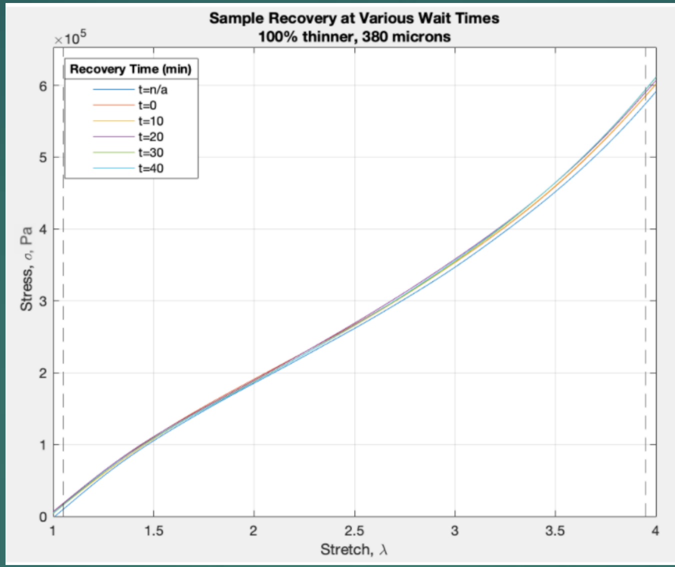




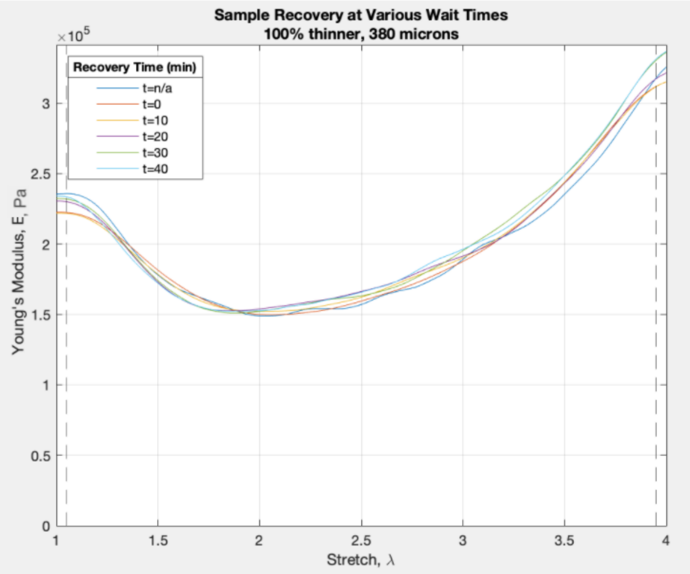


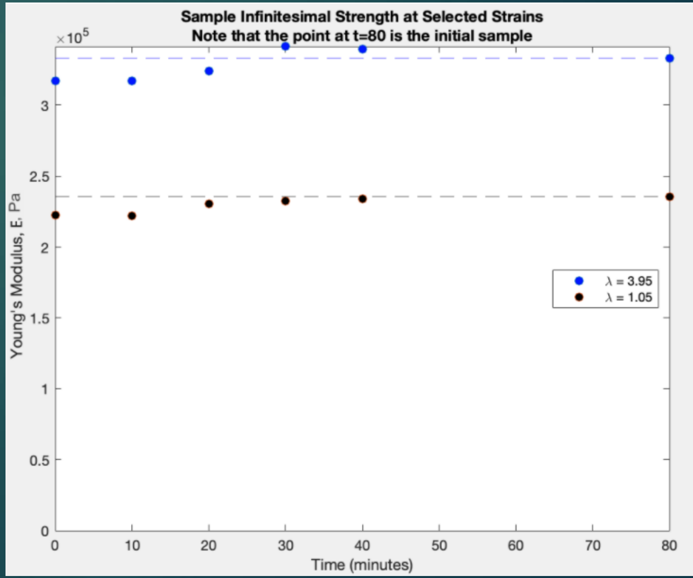
Establishing Sample Wait Time





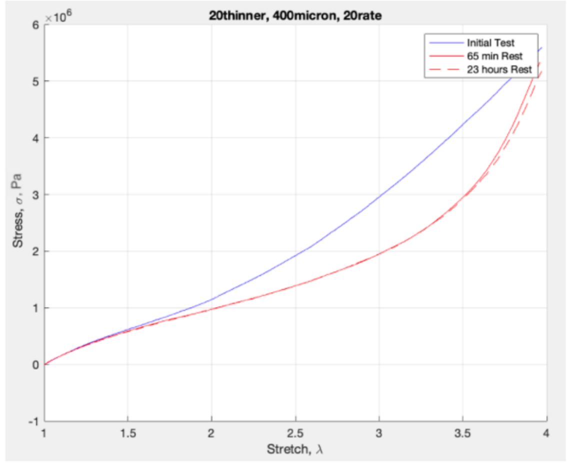
Derive



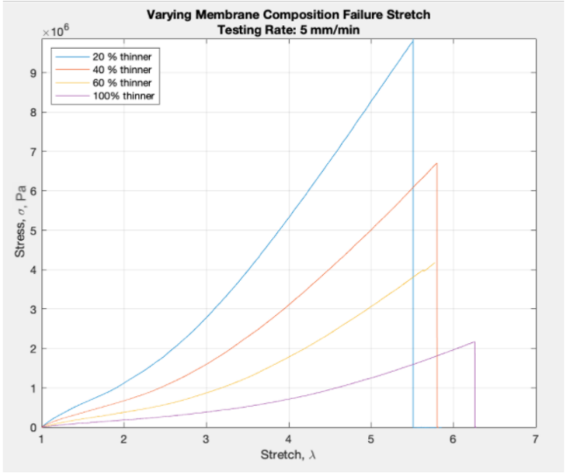


Developing Sample Testing Method

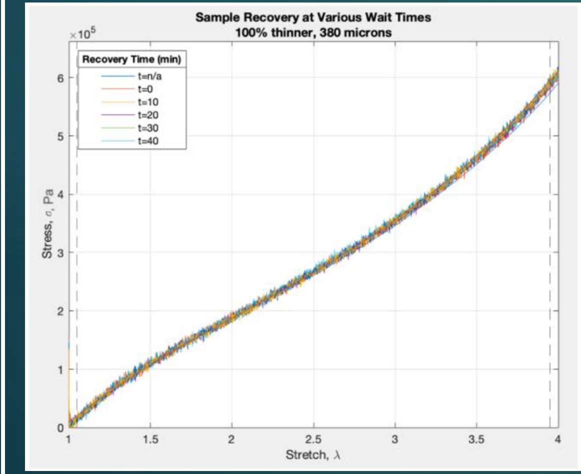
- Need to establish a procedure for uniform testing to eliminate this bias in samples and account for viscoelastic effects
  - Identify appropriate stretch range for repeated testing of samples
  - Account Mullins Effect in samples stretched for the first time
  - Establish wait time to account for Permanent set



Example of Mullins Effect

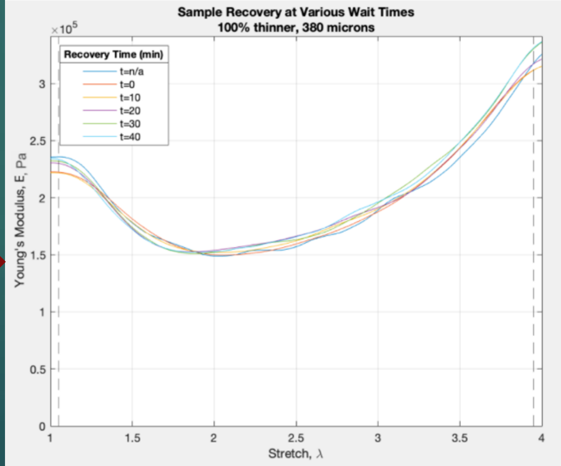


Sample Stretch until Failure

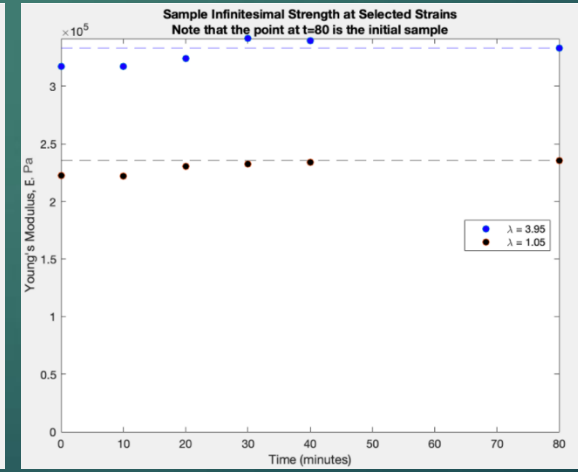


Raw Data

Fit with smooth spline and Derive



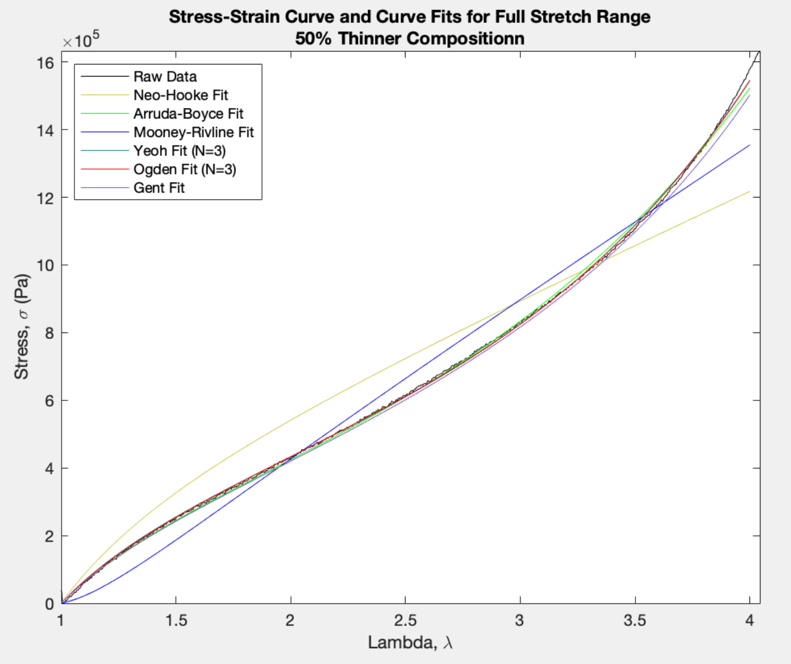
Estimated Tangent Modulus

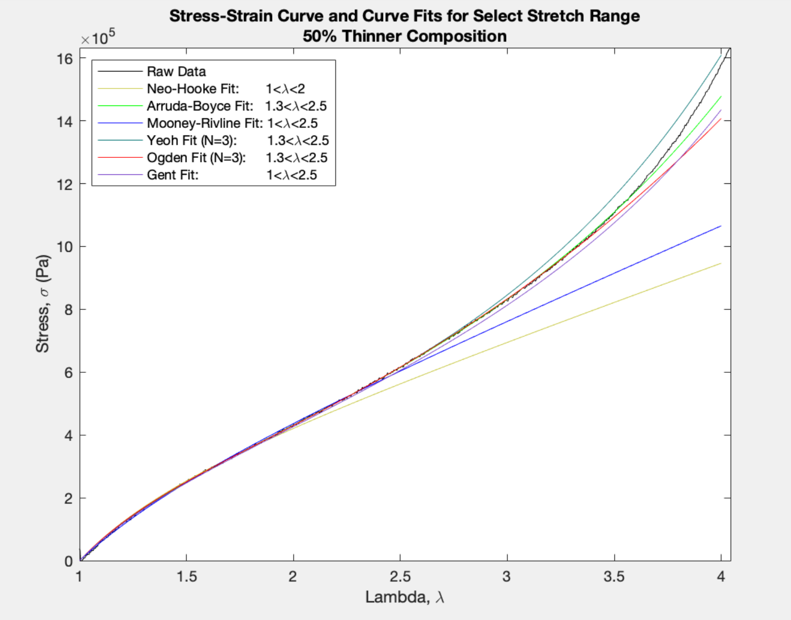


Sample Tangent Modulus at start and end stretch

Hyperelastic Material Modeling: Estimating Shear Modulus

- Various hyperelastic models can be used to fit uniaxial data and obtain a value for shear modulus
- Neo-Hooke, Mooney-Rivlin, Arruda-Boyce, Ogden, Yeoh, and Gent models are all considered
- All calculations assumed incompressibility, isotropy, and uniaxial extension
- All models are fit over the full range (top) and an enhanced range for each model (bottom), from 1-2 or 1-2.5





Hyperelastic Material Modeling

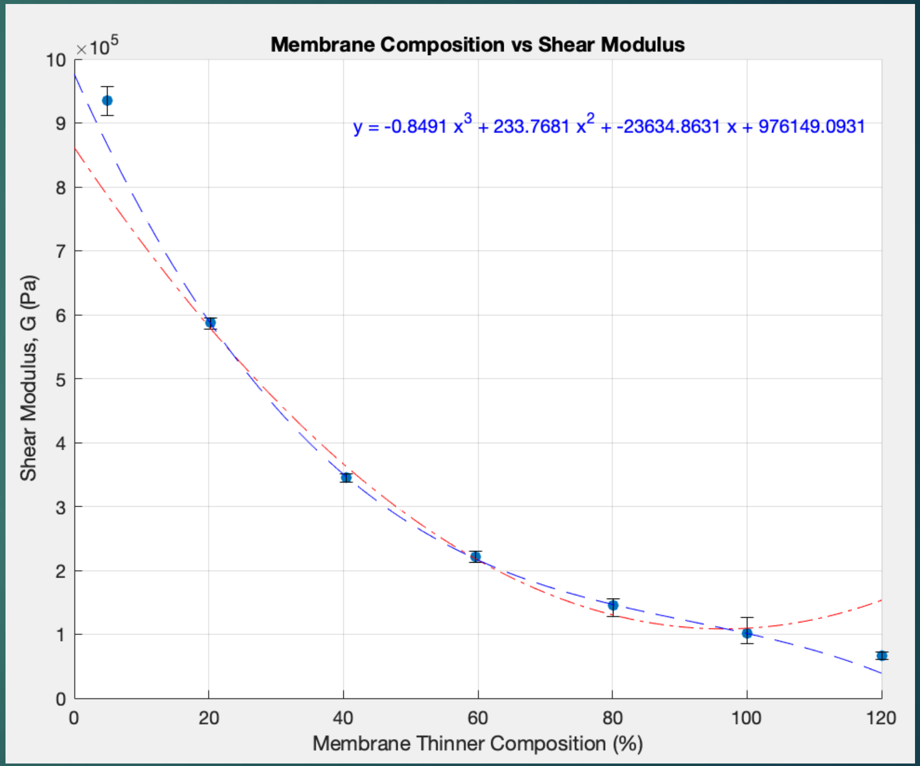
- Each model outputs an estimate for shear modulus based on constants and parameters obtained from curve fits
- The Arruda-Boyce model fit the best and is used as a reference for all models
- Gent model is a simple (2 parameter) model with very high accuracy

Error in Elastic Modulus Estimate between each model and AB model

Hyperelastic Model	20% Thinner Membrane	50% Thinner Membrane	100% Thinner Membrane
Arruda-Boyce (Estimate)	1.68 MPa	0.694 MPa	0.298 MPa
Neo-Hooke	2.38%	3.89%	4.03%
Mooney-Rivlin	1.79%	10.66%	9.06%
Yeoh (N=3)	16.67%	2.59%	3.69%
Ogden (N=3)	5.95%	35.45%	174.50%
Gent	0.60%	0.43%	0.00%

Silicone Polymer Shear Moduli

- Data were fit with Gent model over 1.1-1.5 stretch range to estimate shear modulus and determine a relationship between thinner fraction and material shear modulus
- Membrane thinner composition can now be determined based on a desired shear modulus or Young's modulus
  - E=3G for incompressible materials



Material Modeling

- Neo-Hooke, Mooney-Rivlin, Arruda-Boyce, Ogden, Yeoh, and Gent models were hyperelastic models considered
    - Data was fit over whole range and an optimal range, and material constants are used to obtain an estimate for material shear modulus
- All calculations assumed incompressibility, isotropy, and uniaxial extension
- Gent model ultimately chosen to estimate shear modulus and determine a relationship between thinner fraction and material shear modulus
    - Membrane thinner composition can now be determined based on a desired shear modulus

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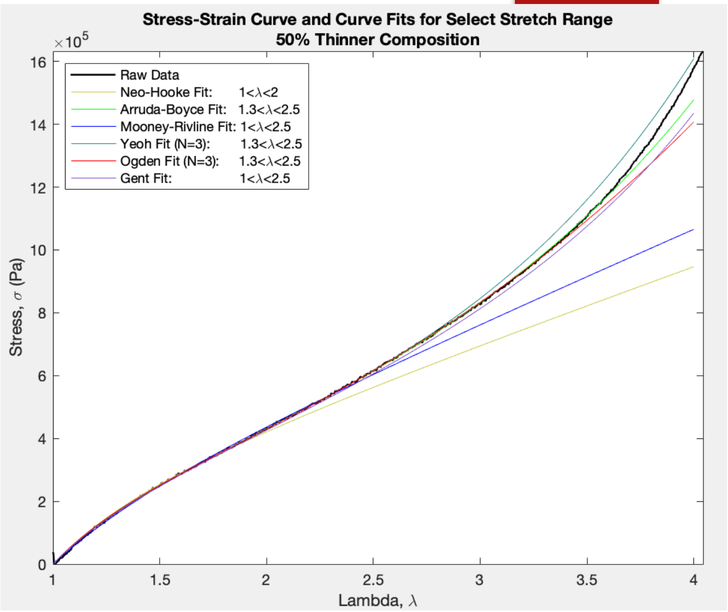
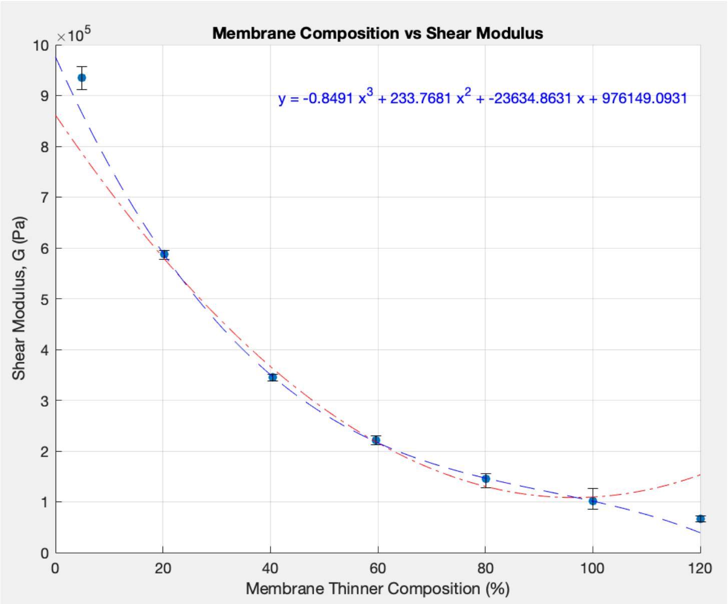
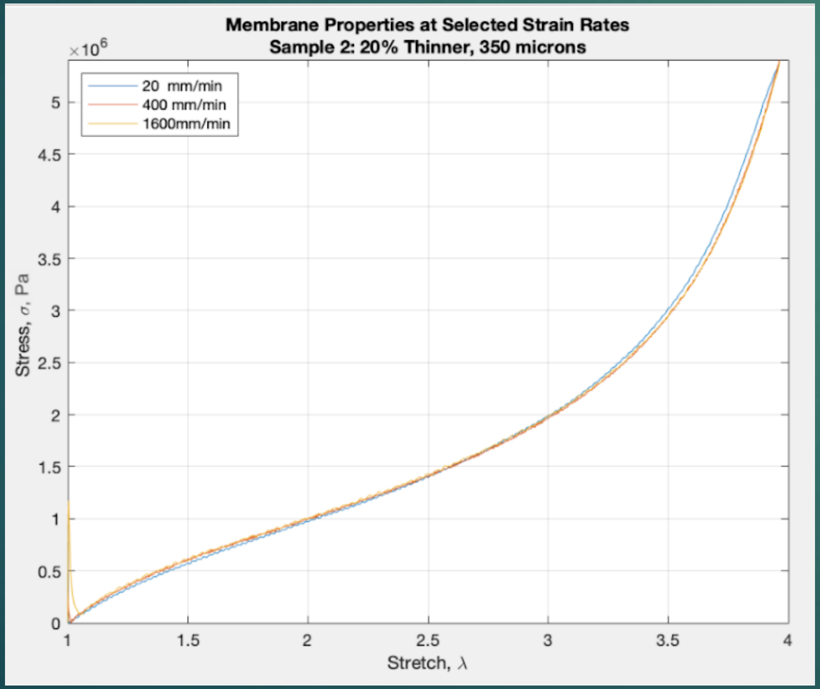
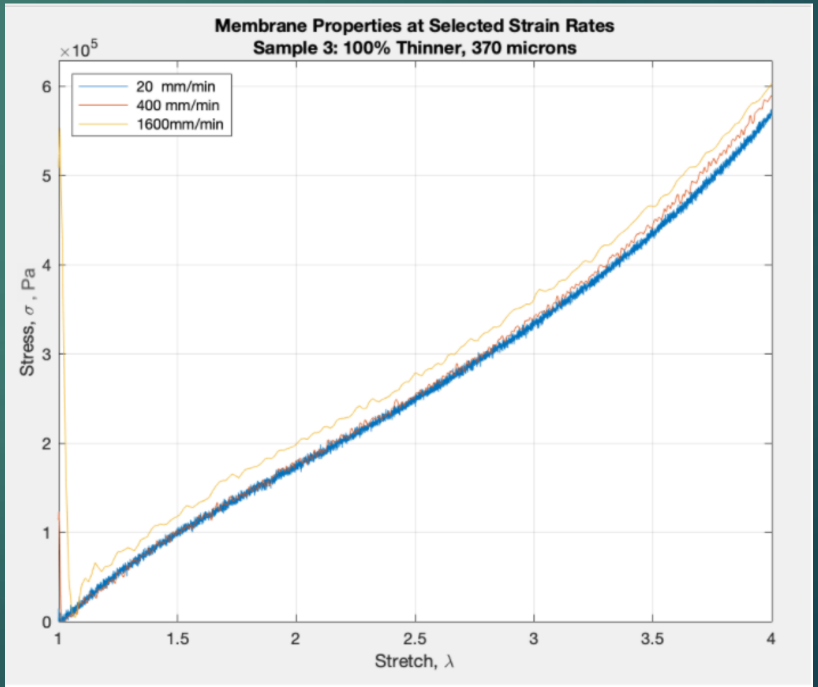




Varying Strain Rate

- Increased strain rate to analyze viscoelastic behavior and see if this plays a potential role in the materials
  - Viscous damping is dependent on a coefficient and the rate of stretching
- No discernible change in material behavior at higher strain rates



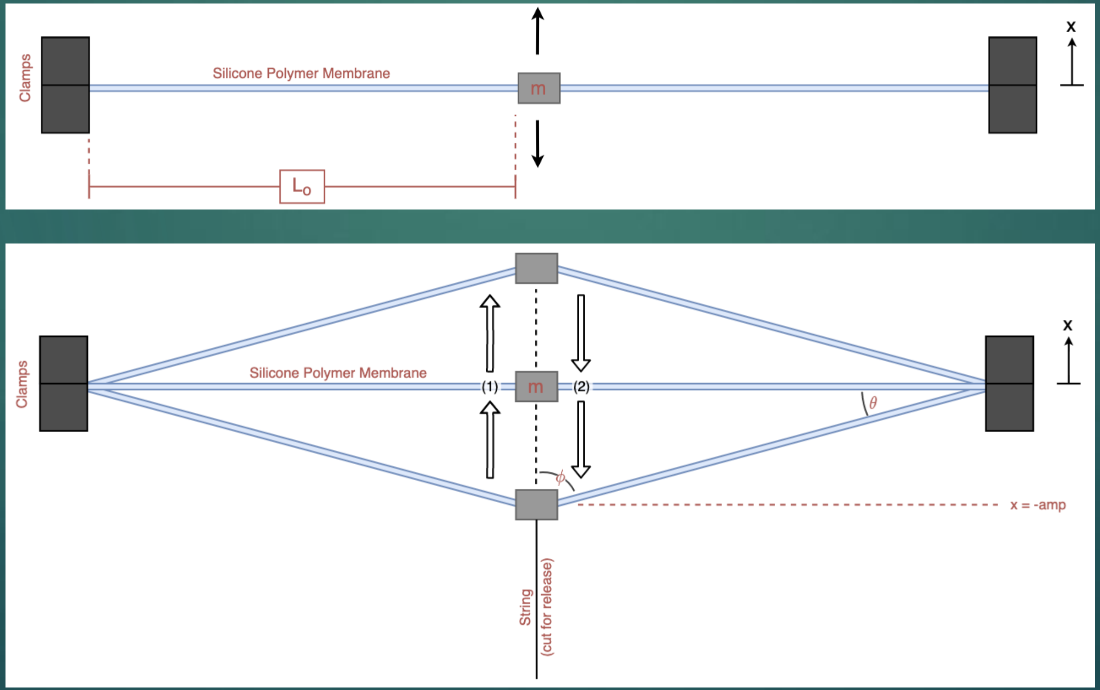


Mechanical Oscillator

- Oscillate mass suspended by silicone sample to estimate material elastic modulus and viscous damping
- Constitutive equation will include an inertial term dependent on acceleration, a restoring force dependent on material stiffness, and a damping term dependent on strain rate
  1. Excite system and record time-dependent motion
  2. Use MATLAB video tracking to plot position with time
  3. Compare MATLAB tracking with predicted motion from equation of motion
  4. Use ring-down method to estimate viscous damping coefficient

Mechanical Oscillator Set-Up

- Mechanical oscillator configuration is chosen based on two primary criteria:
  - Straightforward set-up in which mass can be easily record and tracked
  - Material always remaining in tension
- Horizontal configuration is selected and shown below



Governing Equations and Equation of Motion

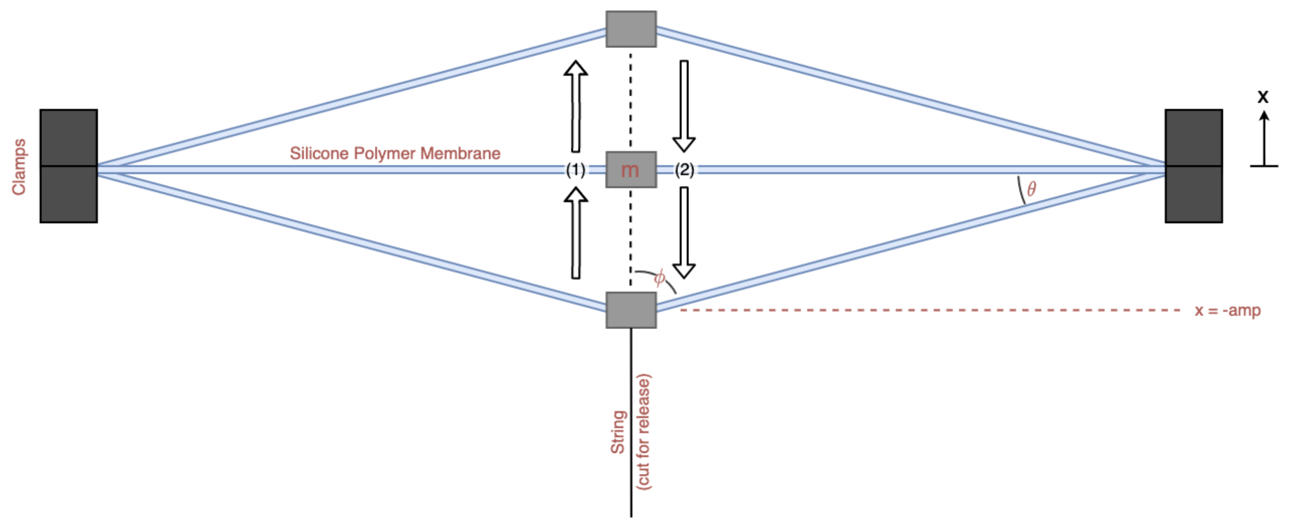
- Governing equation is derived from Newton’s Law with three contributions to force govern motion
- ODE45 is used to solve the system on MATLAB given initial displacement and velocity
- This predicted motion is compared to experimental results and subsequently used to estimate E
- E is assumed constant (linear elastic) for small stretch ranges covered during oscillation

$$F = m\frac{d^2x}{dt^2}$$

$$F = F(\varepsilon) + F(\eta, \frac{d\varepsilon}{dt}) + F(g)$$

$$m\frac{d^2x}{dt^2} = -2EA_o sin(\theta)(\frac{(x^2+L_i^2)^{1/2} - L_o}{L_o}) - 2\eta A_o(\frac{x}{(x^2+L_i^2)^{1/2} L_0})\frac{dx}{dt}cos(\phi) - mg$$

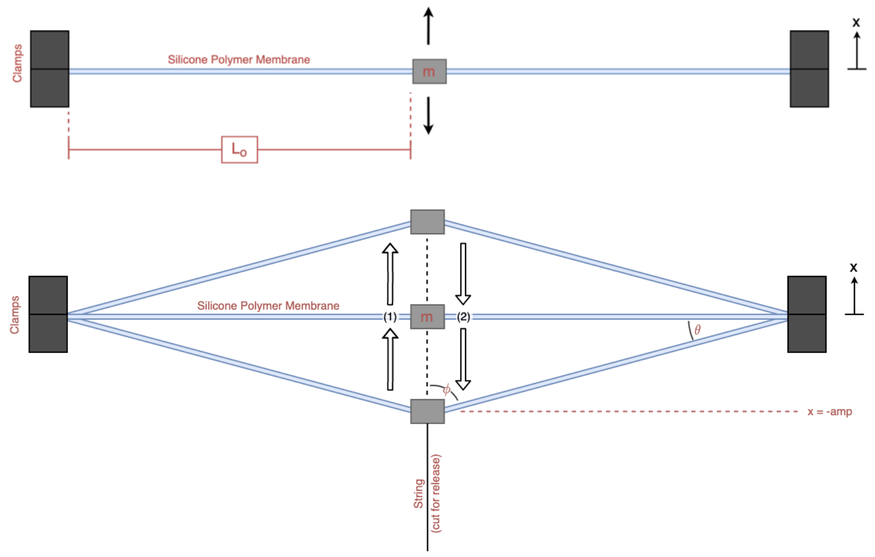
$$\theta = tan^{-1}(\frac{x}{L_i}) \text{ and } \phi = tan^{-1}(\frac{L_i}{x})$$



Mechanical Oscillator

- Oscillate mass suspended by silicone sample to estimate material elastic modulus and viscous damping
- Configuration based on criteria that the material must always be in tension and mass must be easily recorded and tracked by a camera
- Constitutive equation will include an inertial term dependent on acceleration, a restoring force dependent on material stiffness, and a damping term dependent on strain rate
  1. Excite system and record time-dependent motion
  2. Use MATLAB video tracking to plot position with time
  3. Compare MATLAB tracking with predicted motion from EOM (determined from MATLAB ODE solver)
  4. Use ring-down method to estimate viscous damping coefficient

*Linear elasticity assumed over small stretch ranges (E is constant)



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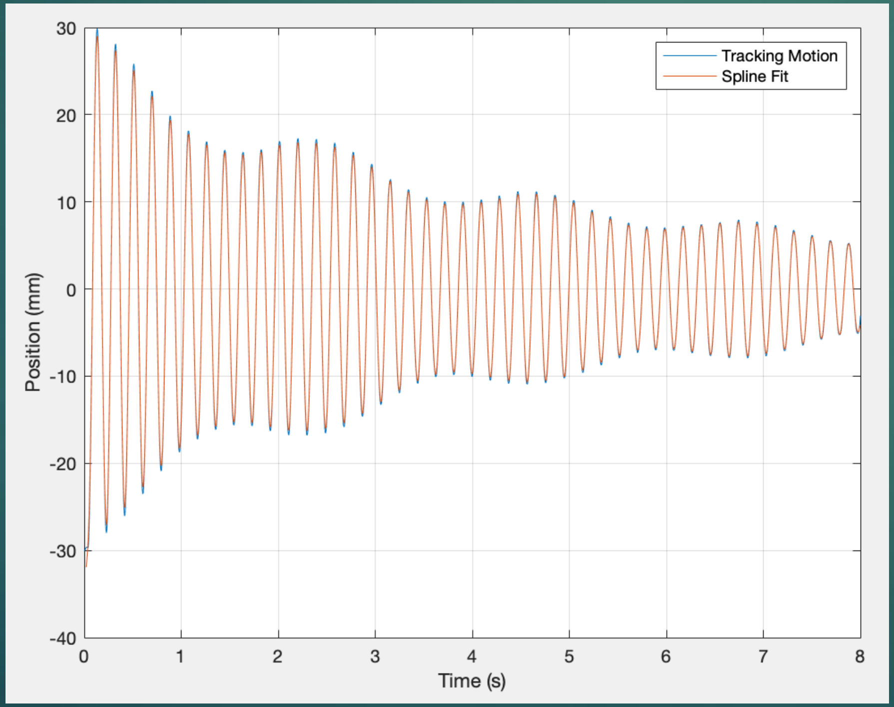
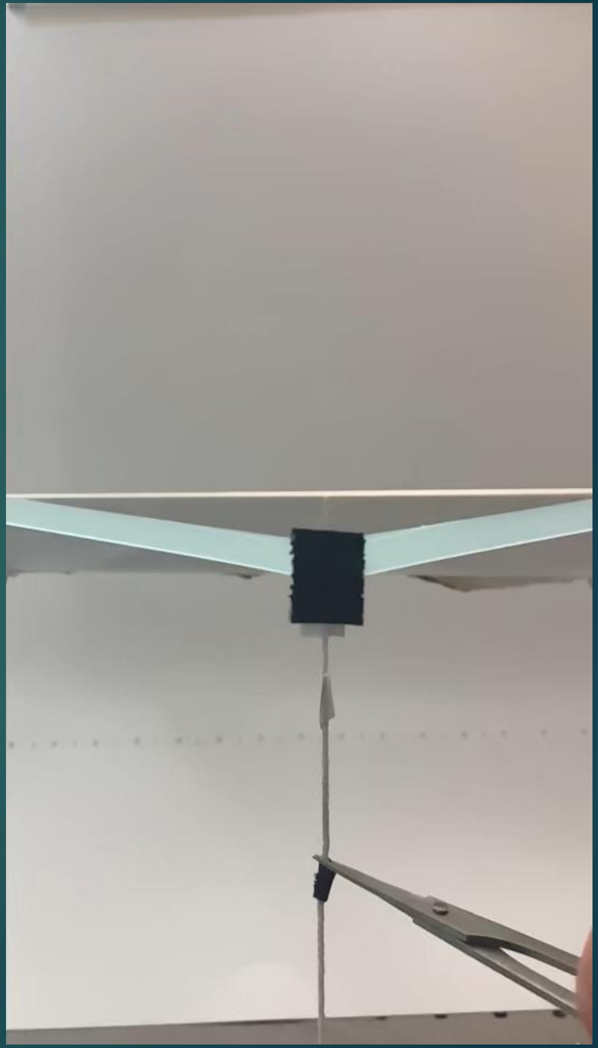
$$\theta = tan^{-1}(\frac{x}{L_i}) \text{ and } \phi = tan^{-1}(\frac{L_i}{x})$$

Experimental Testing

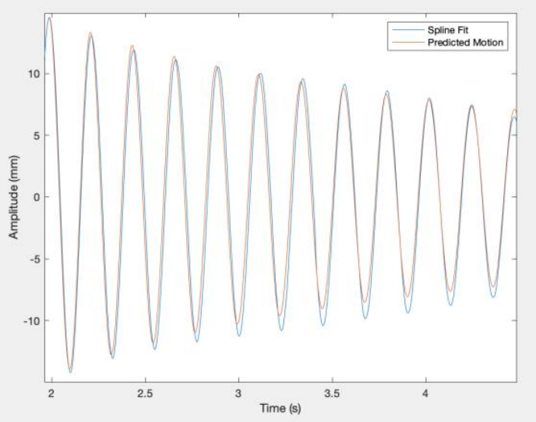
- Videos recorded in slow motion with an iPhone
  - 240 fps, playback at 1/8th speed
- Black tape attached to mass to make MATLAB tracking easier
- Spline is fit for smooth data



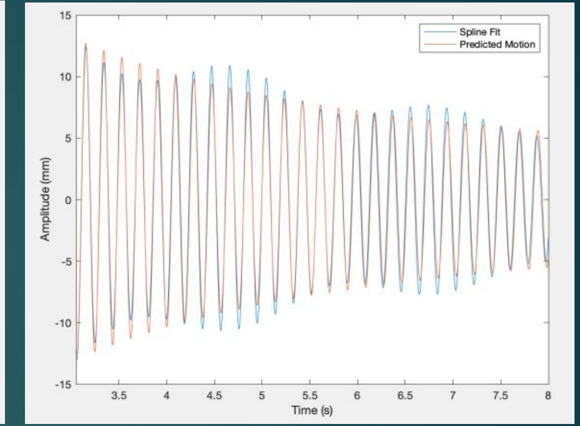


Results

- Gravity is negligible – predicted motion is only accurate when gravitational force is ignored. Oscillations are dominated by inertial force and elastic force
- Period ranges from 0.23 to 0.15 s
- E can be effectively estimated by matching predicted motion with experimental data



λ=1.1



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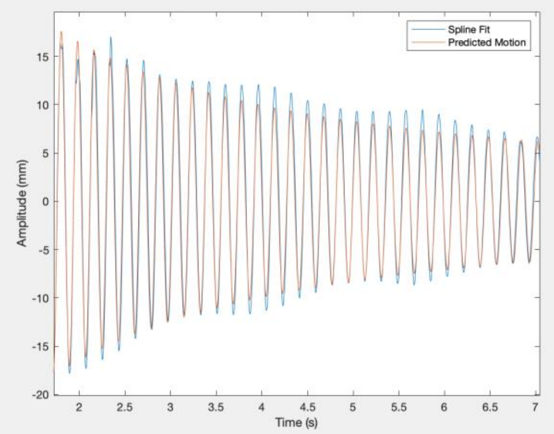
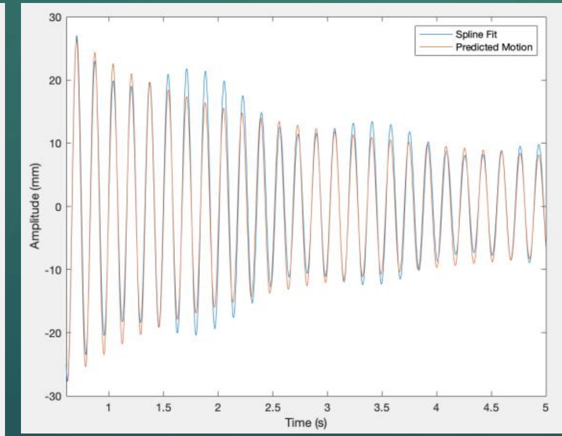
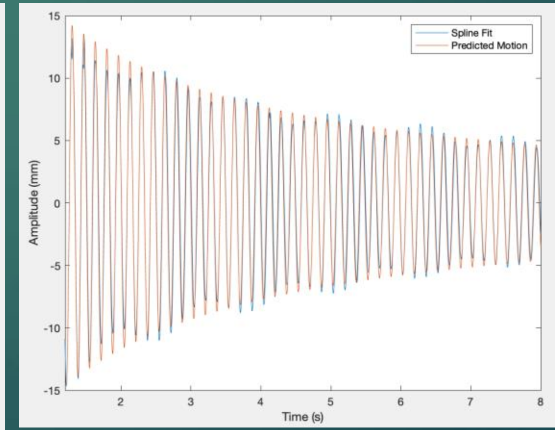


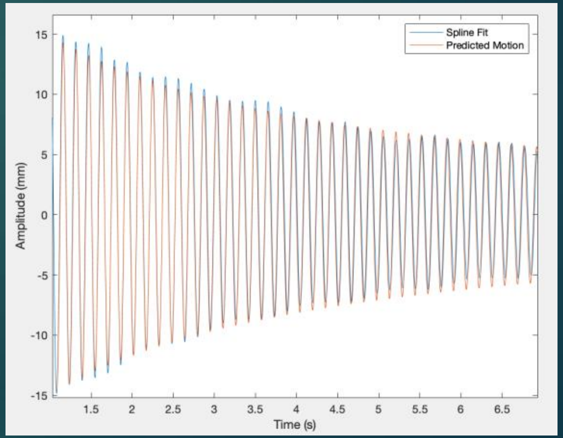
λ=1.4



λ=1.5



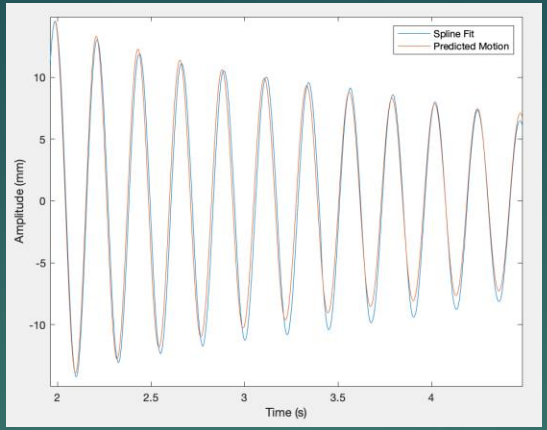
λ=1.6



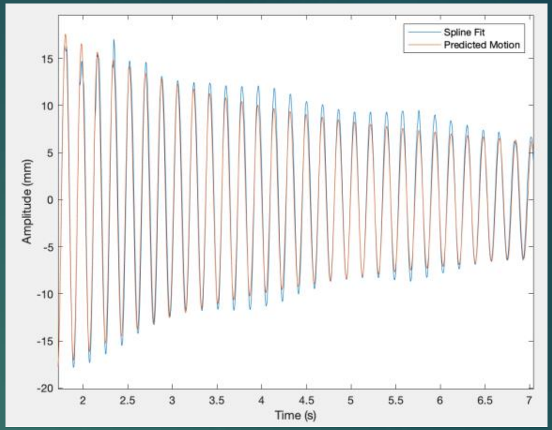
λ=1.8

Results

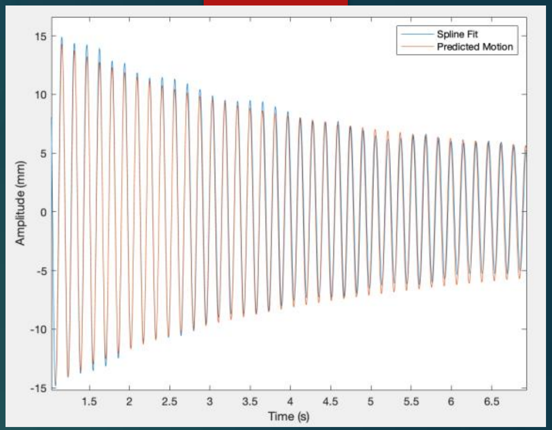
- Gravity is negligible
- E is estimated by matching predicted motion with experimental data
- Damping is determined by fitting amplitudes to a curve with the form $y = Ae^{-bt}\cos(\frac{2\pi}{T}t)$ where T is period and $b = \frac{\eta}{2m}$
  - $y = Ae^{-bt}$ characterizes damping/amplitudes



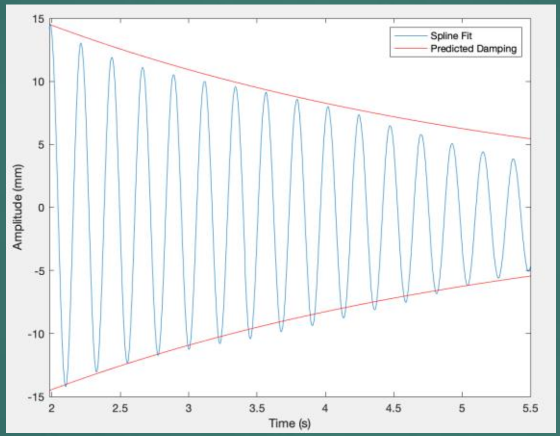
λ=1.1



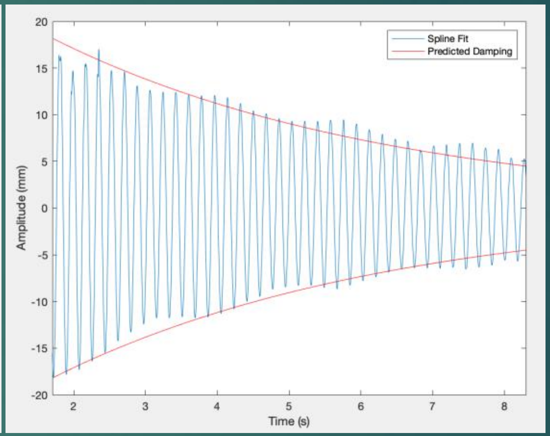
λ=1.4



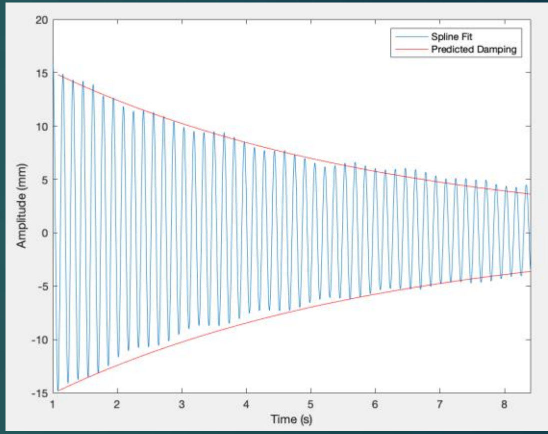
λ=1.8



λ=1.1



λ=1.4



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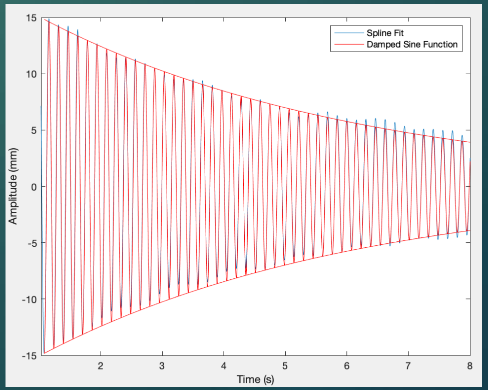
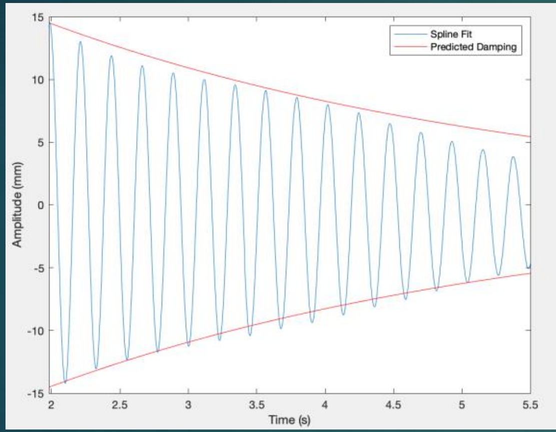


Results

- Damping is determined by fitting amplitudes to a curve with the form $y = Ae^{-bt}$ where $b = \frac{\eta}{2m}$
- Similarly, the motion can be fit by $y = Ae^{-bt}\cos(\frac{2\pi}{T}t)$ where T is period



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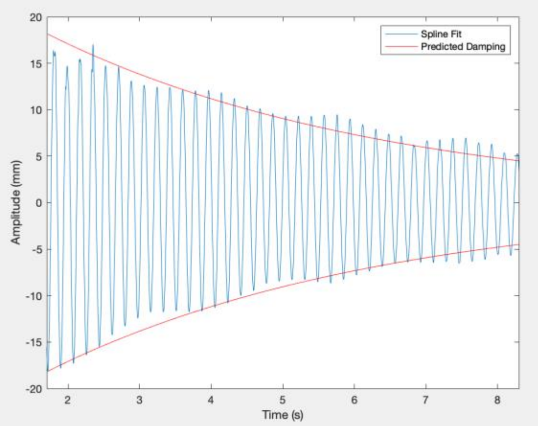
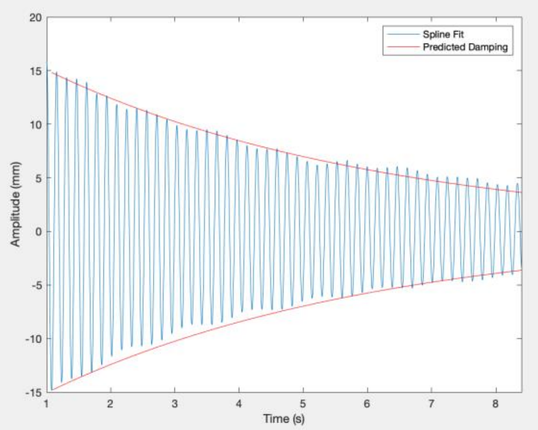


λ=1.4



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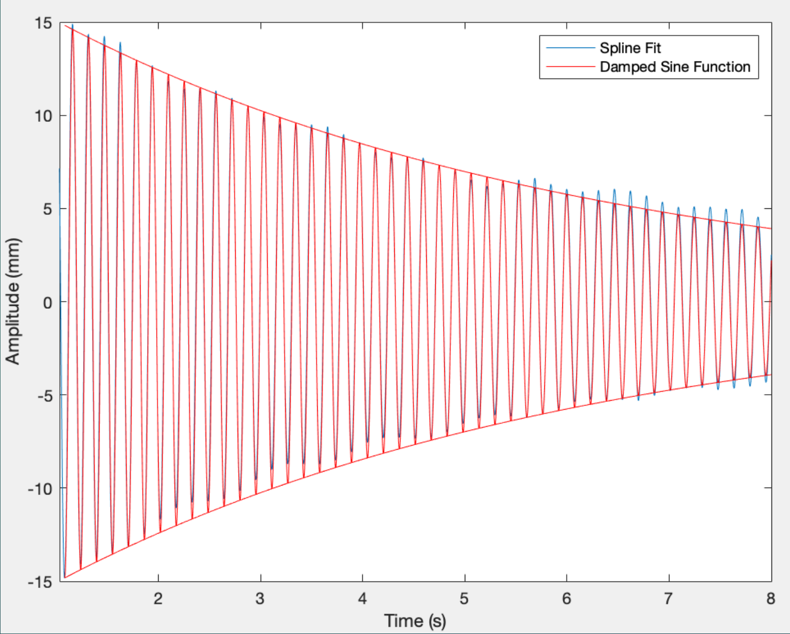

Stretch Range	Period (s)	Strain Rate (s^{-1})	Elastic Modulus (MPa)	Elastic Modulus Error (Min-Max)	Damping Coefficient (10^{-3} Ns/m)	Damping Uncertainty
1.10-1.19	0.226	1.68	0.224	0.215 - 0.234	14.5	±3.5
1.20-1.26	0.205	1.26	0.213	0.209 - 0.224	11.0	±2.0
1.30-1.36	0.189	1.27	0.190	0.180 - 0.198	12.0	±2.5
1.40-1.46	0.180	1.25	0.176	0.169 - 0.183	10.5	±2.0
1.50-1.59	0.170	2.16	0.170	0.162 - 0.174	13.0	±3.0
1.60-1.62	0.166	0.53	0.159	0.152 - 0.166	11.5	±2.5
1.70-1.72	0.161	0.52	0.152	0.145 - 0.159	10.5	±2.5
1.80-1.82	0.156	0.50	0.152	0.142 - 0.156	10.0	±0.5
1.90-1.92	0.149	0.50	0.153	0.145 - 0.159	11.5	±2.5

- Results are from 500 micron sample with 75mm length, 80mm width, and 26.0g total mass
- Damping uncertainty based on differences in amplitudes from secondary frequencies
- Elastic modulus error based on uncertainties in measurements and differences in tracking results for multiple samples

Results

ELASTIC MODULUS AND DAMPING COEFFICIENTS FROM OSCILLATOR DATA

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ELASTIC MODULUS AND DAMPING COEFFICIENTS FROM OSCILLATOR DATA

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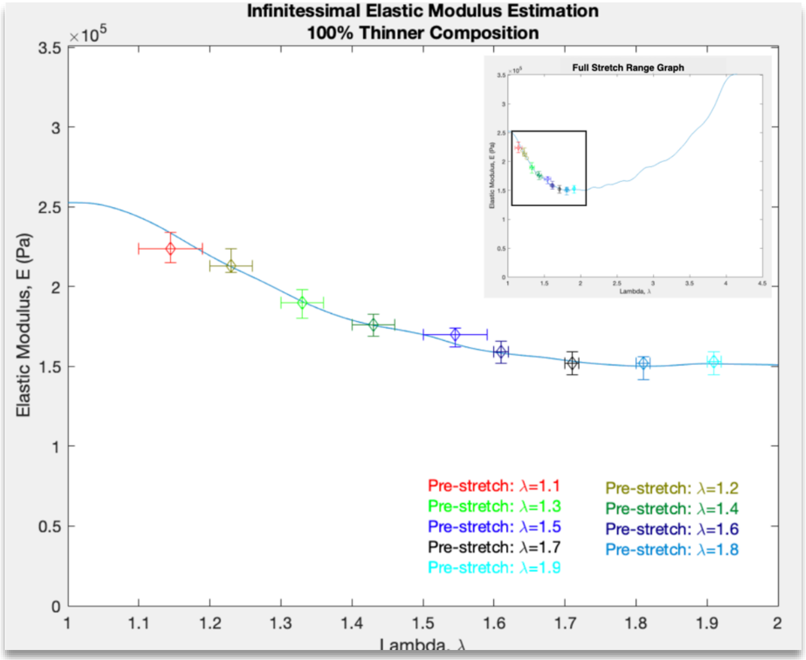
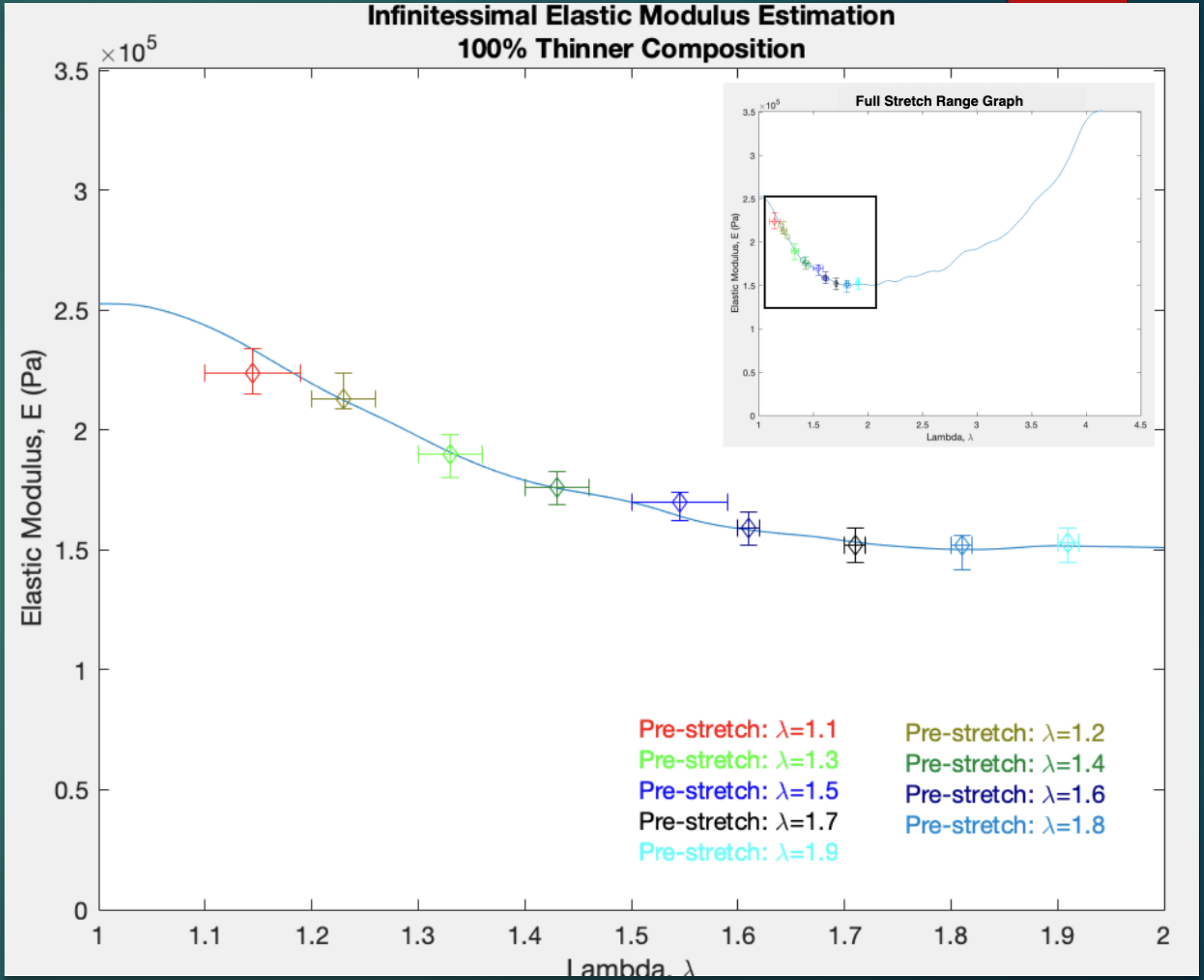
1.30-1.36	0.189	1.27	0.190	0.180 - 0.198	12.0	±2.5
1.40-1.46	0.180	1.25	0.176	0.169 - 0.183	10.5	±2.0
1.50-1.59	0.170	2.16	0.170	0.162 - 0.174	13.0	±3.0
1.60-1.62	0.166	0.53	0.159	0.152 - 0.166	11.5	±2.5
1.70-1.72	0.161	0.52	0.152	0.145 - 0.159	10.5	±2.5
1.80-1.82	0.156	0.50	0.152	0.142 - 0.156	10.0	±0.5
1.90-1.92	0.149	0.50	0.153	0.145 - 0.159	11.5	±2.5



- Results are from 500 micron sample with 75mm length, 80mm width, and 26.0g total mass
- Damping uncertainty based on differences in amplitudes from secondary frequencies
- Elastic modulus error based on uncertainties in measurements and differences in tracking results for multiple samples

Comparison to Uniaxial Estimates

- Elastic modulus predicted from oscillator are shown with diamond markers and error bars
- Elastic modulus determined from uniaxial testing shown by circular markers & blue line



Takeaways and Conclusion

- Uniaxial studies show promise: silicone polymer material is appropriate for this application
- Hyperelastic models: Gent model is very impressive given its simplicity and can be used to obtain shear modulus estimate from uniaxial testing
  - Arruda-Boyce model is slightly more complex but also very effective
- Mechanical oscillator has potential in predicting material properties and could be a low-cost alternative to uniaxial machines
  - Repeated testing with different pre-stretch can recreate stress-strain relationship and estimate E at a given stretch
  - Horizontal configuration damping is likely caused by air resistance. Vertical configuration could be investigated to estimate damping, although large strain variance in this configuration can not be modeled by a constant E
- More complex behavior than initially thought: Damped oscillator EOM actually predicts variable damping coefficient at varying pre-stretch

Takeaways and Conclusion

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- Hyperelastic models: Gent model is very impressive given its simplicity and can be used to obtain modulus estimate from uniaxial testing
  - Arruda-Boyce model is slightly more complex but also very effective
- Mechanical oscillator has potential in predicting material properties and could be a low-cost, low-tech alternative to uniaxial machines
  - Repeated testing with different pre-stretch can recreate stress-strain relationship and estimate E at a given stretch
  - Further study is required to investigate the source of damping. Damping could be the result of air resistance in this configuration

Thank you!